Developing simple physical descriptions of stagnation in the presence of non-radial flows

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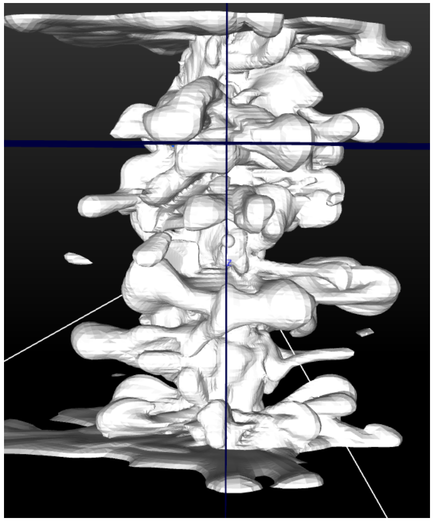
NISP workshop
March 8-9
Livermore, CA

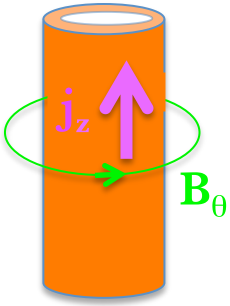
Making connections between 3D simulation and 1D theory is beneficial

3D

(Z pinch)



1D



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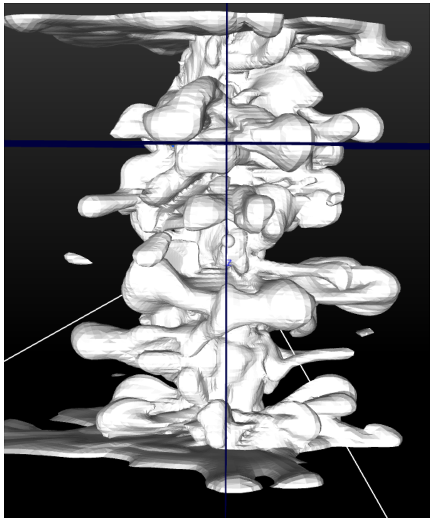

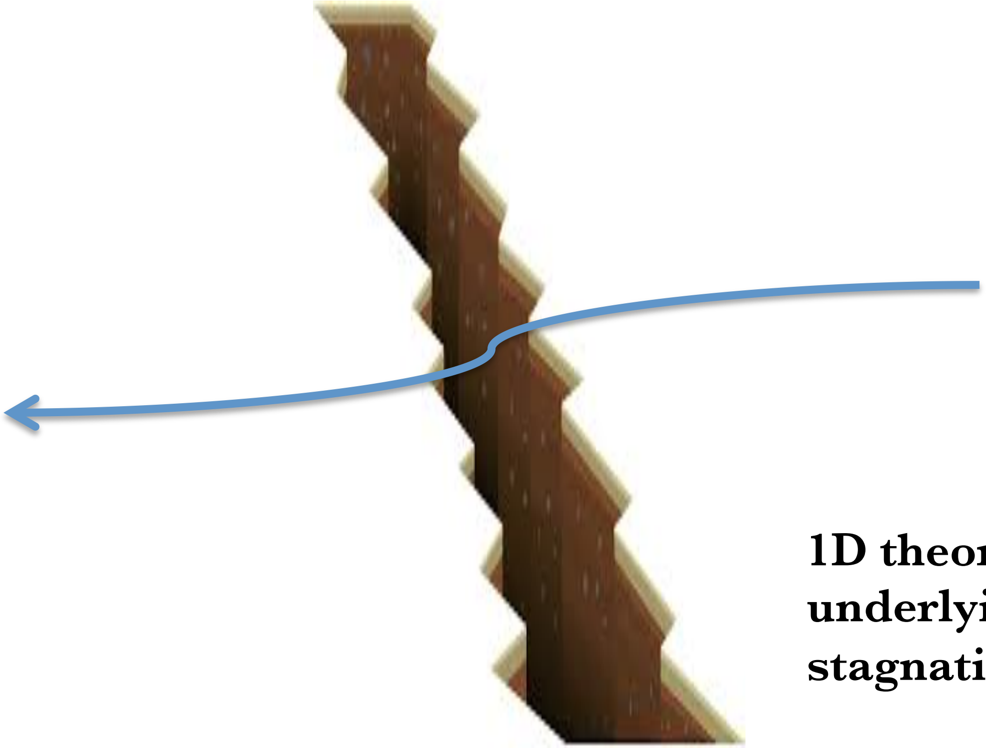

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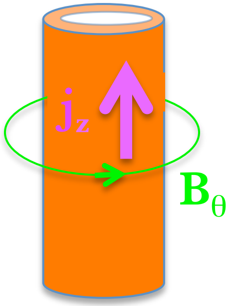

1D theory can give an underlying picture of the stagnation dynamics.

Making connections between 3D simulation and 1D theory is beneficial

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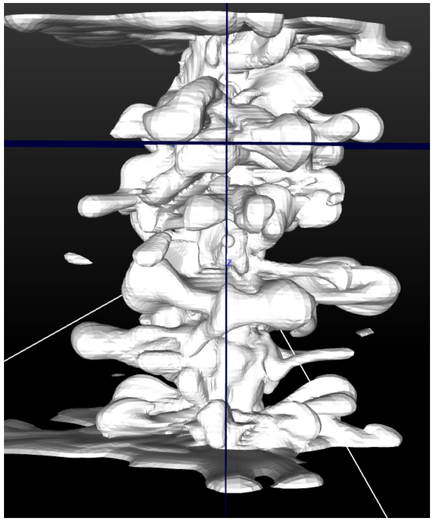

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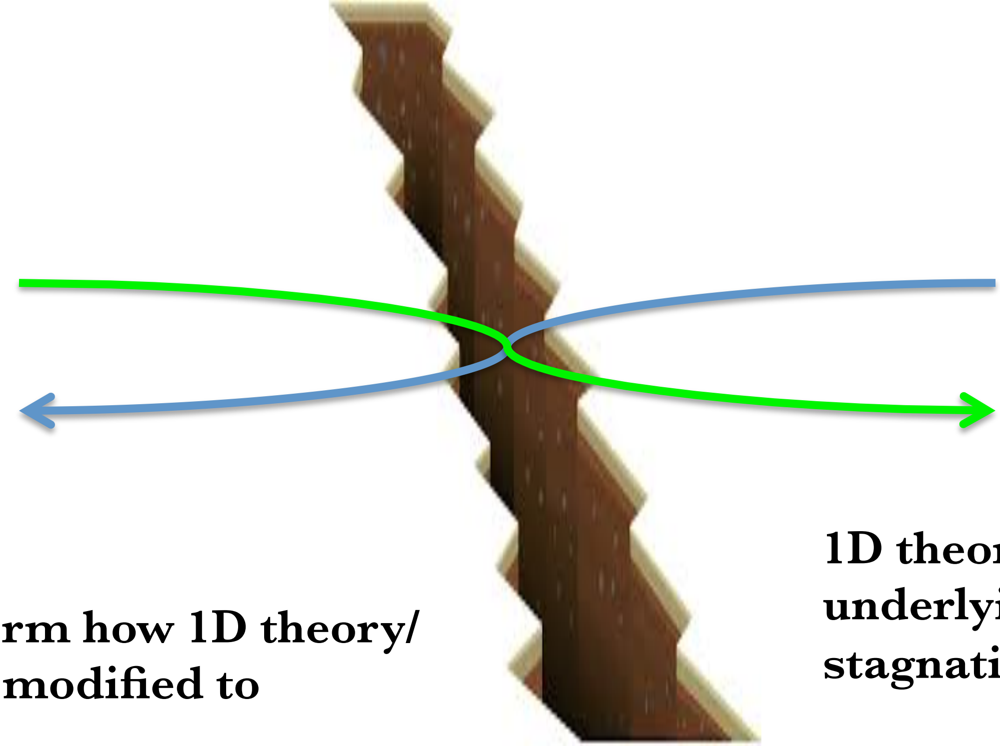
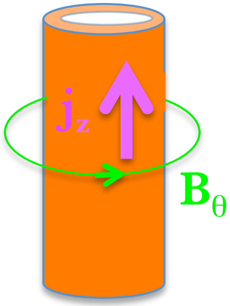




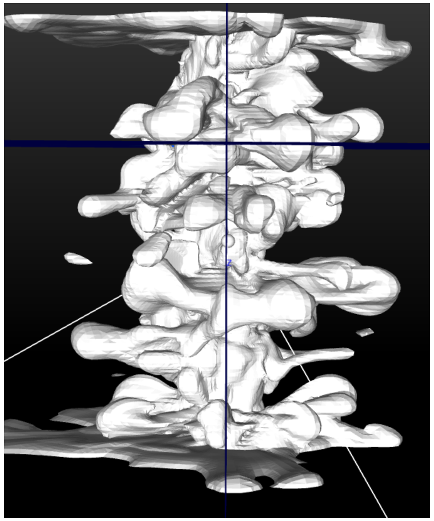
3D simulation can inform how 1D theory/simulations should be modified to account for 3D effects.

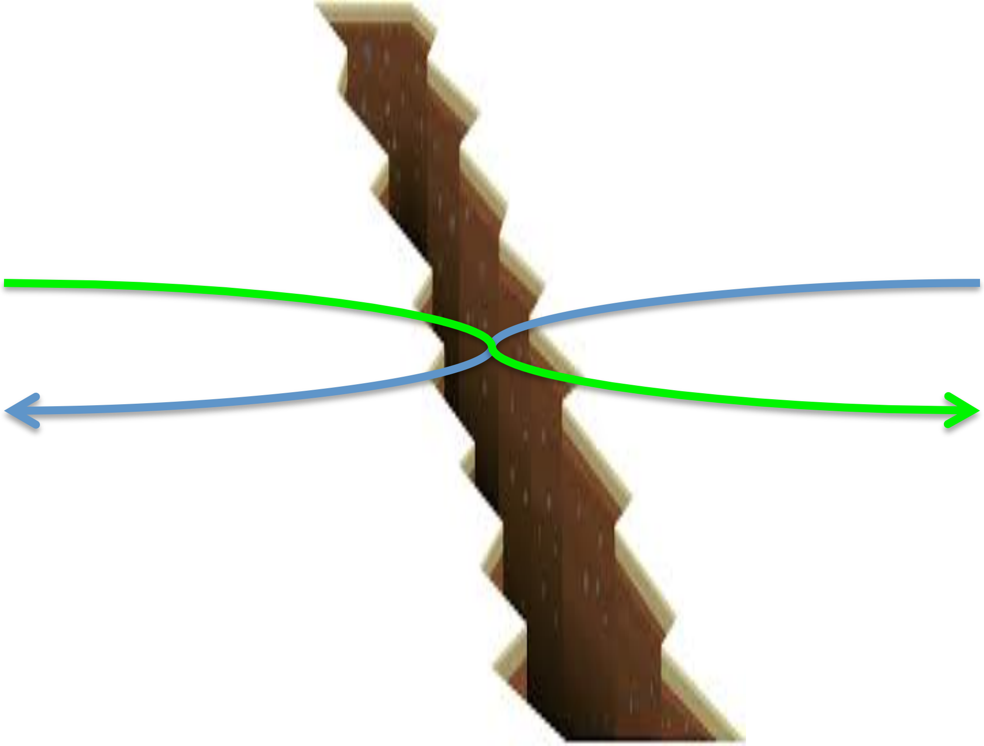
1D theory can give an underlying picture of the stagnation dynamics.

Making connections between 3D simulation and 1D theory is beneficial

3D

(Z pinch)





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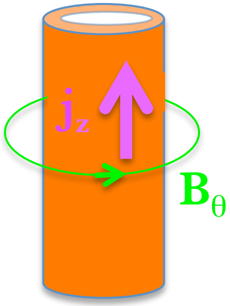
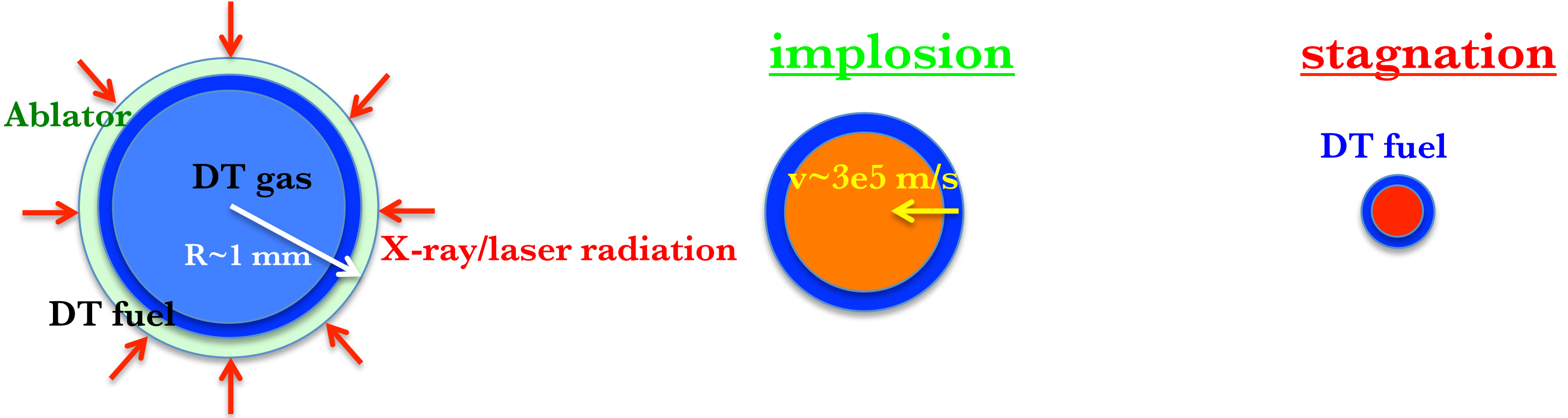


Questions:

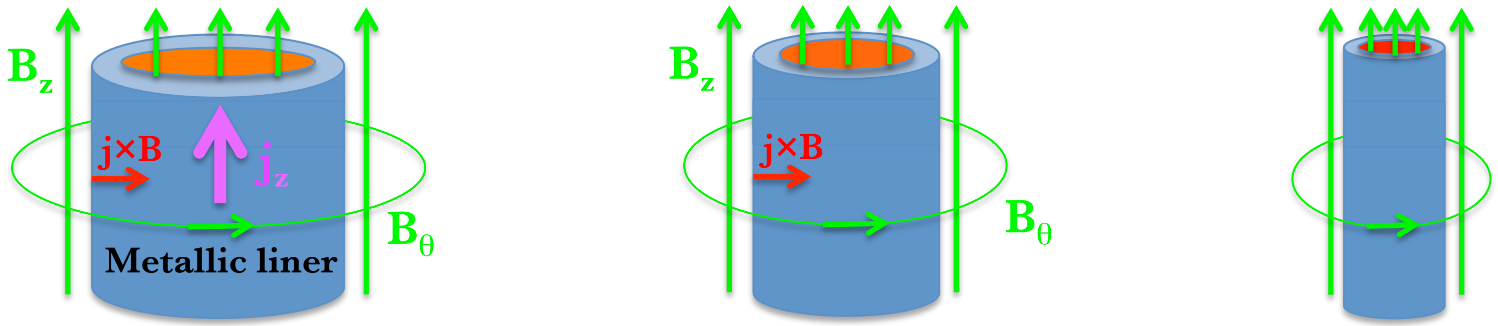
1. What 1D physics is important in driving residual flow at stagnation?
2. Effects of residual flow on pressure/energy balance at stagnation?
3. Can a 1D model approximately describe 3D stagnation?

Z pinches use implosion/stagnation to achieve high energy density (HED)

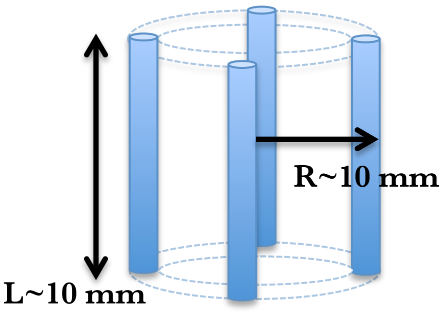
Spherical: ICF capsule



Cylindrical: MagLIF (ICF)



Cylindrical: wire array Z pinch (radiation source)



These arrays constitute hundreds of tiny wires (R_w~5 μm), although I am showing only 4.

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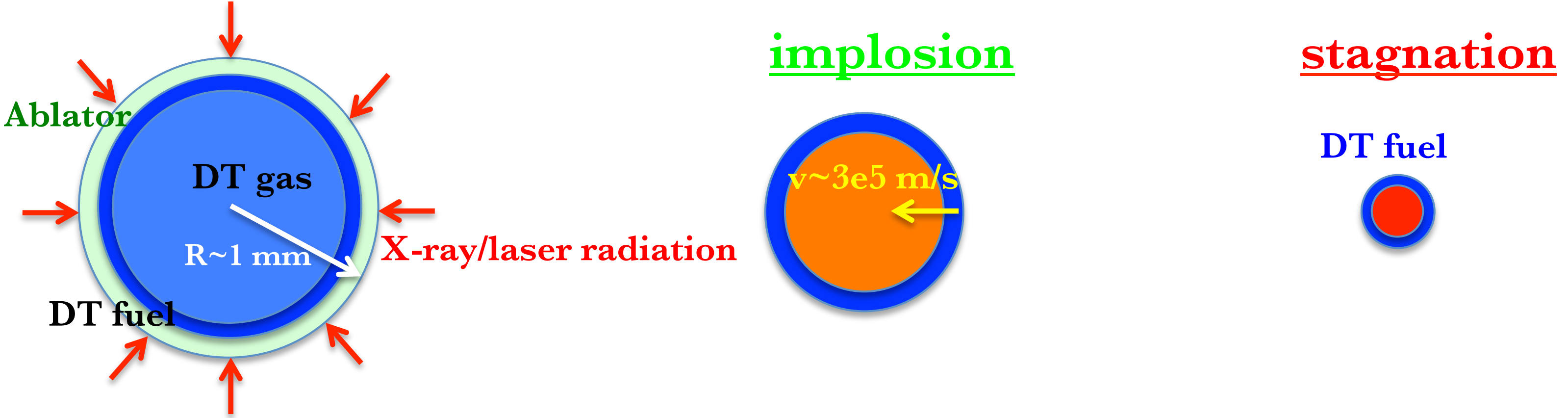

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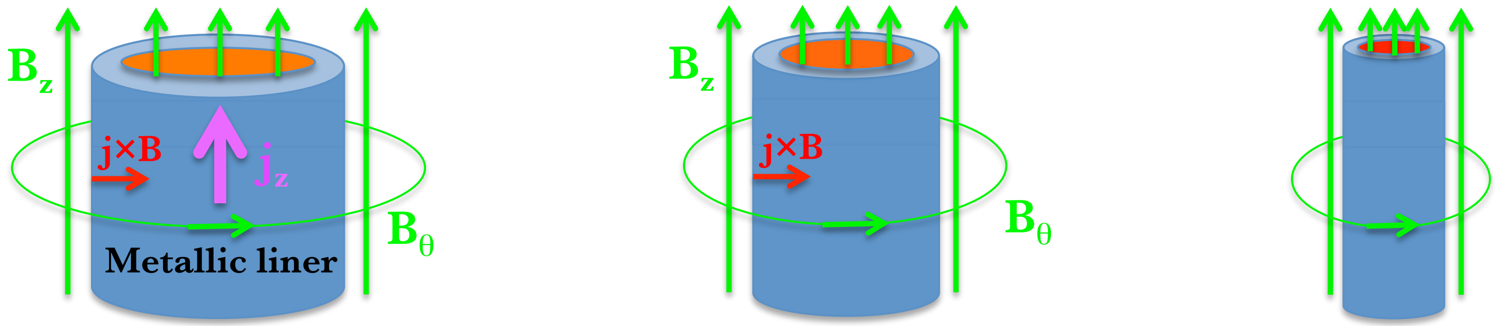

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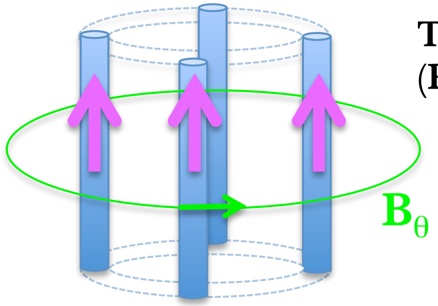

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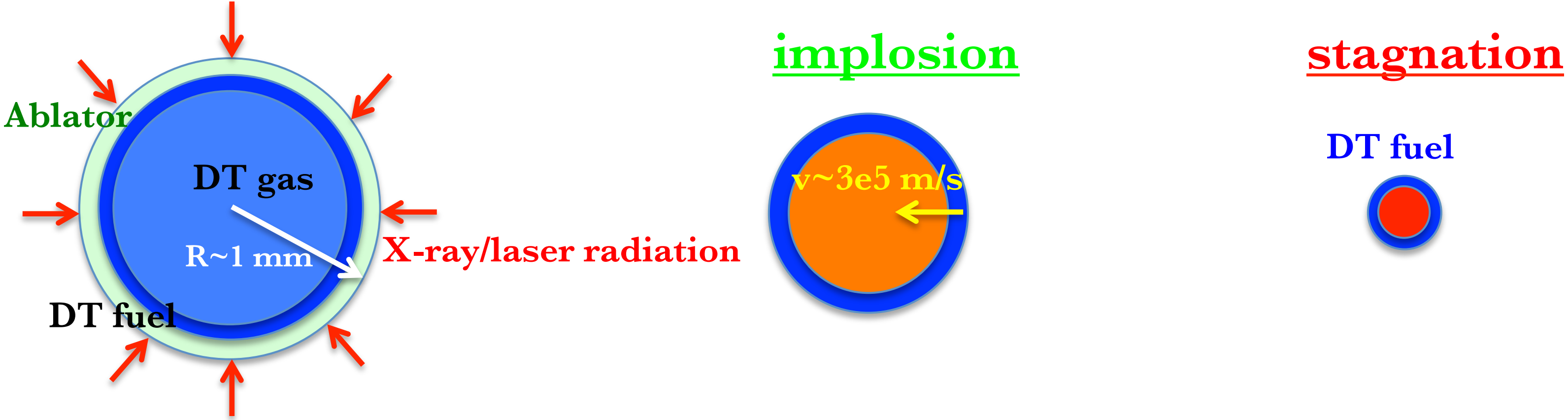

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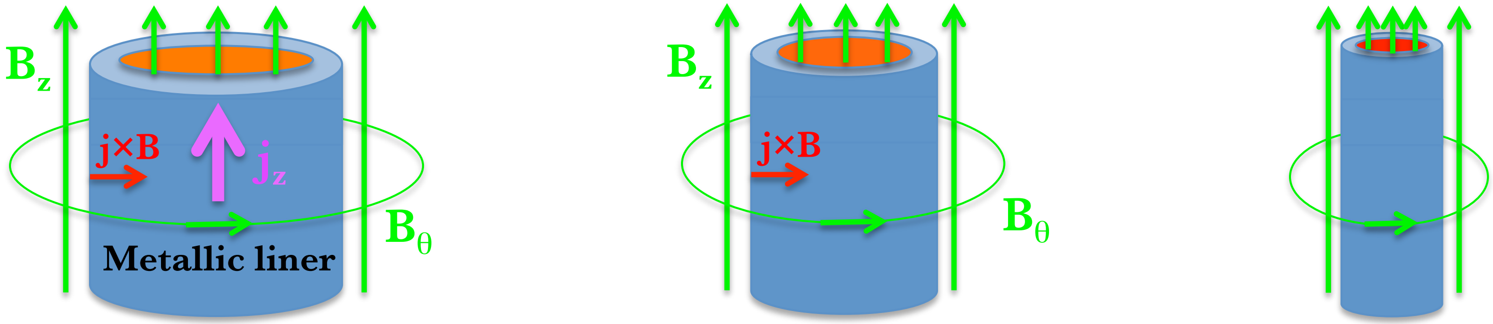

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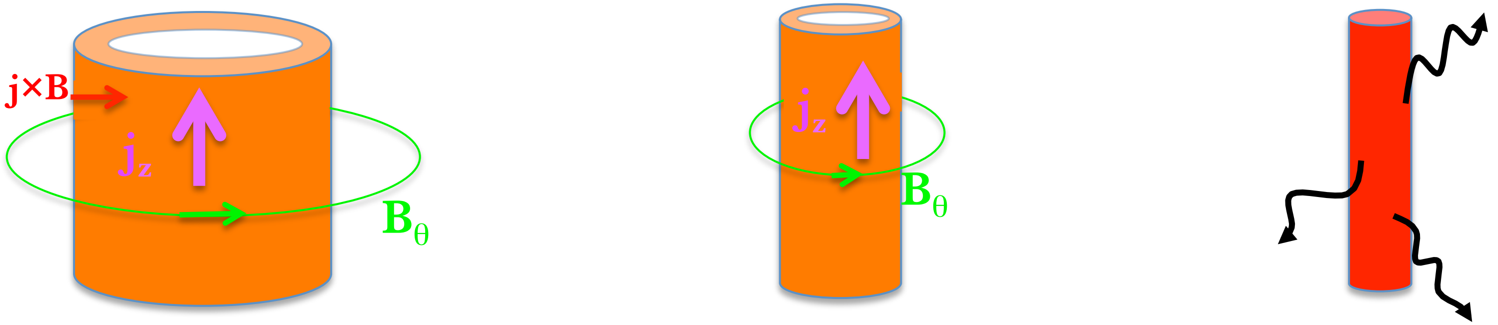


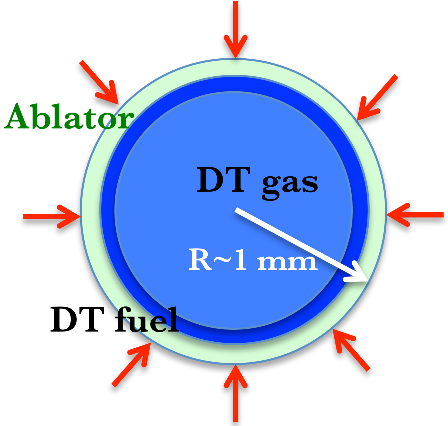
This is a gross oversimplification

For more details: M.G. Haines, Plasma Phys. Control. Fusion **53**, 093001 (2011)

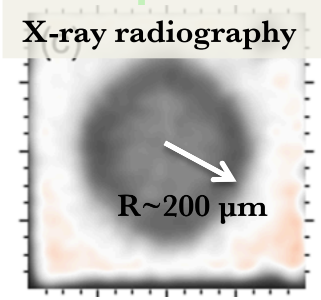
Like ICF capsules/MagLIF, the Z pinch is 3D

3D Simulation

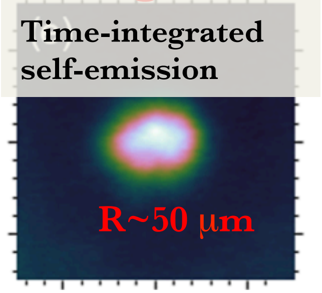
Spherical: ICF capsule



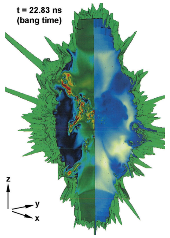
implosion



stagnation



D.S. Clark et al., PoP 22, 022703 (2015)



J.R. Rygg et al., PRL **112**, 195001 (2014)

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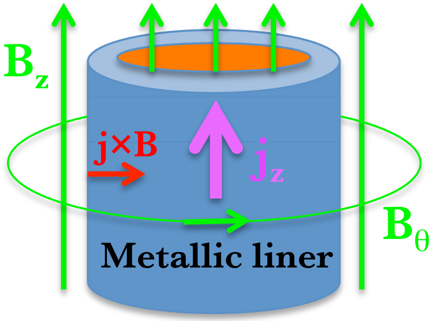
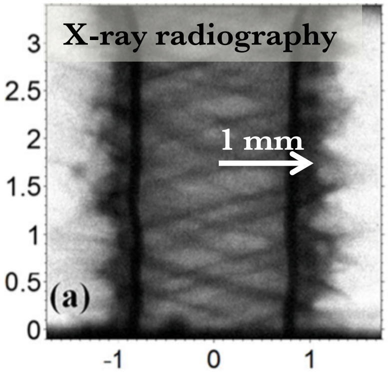
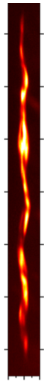




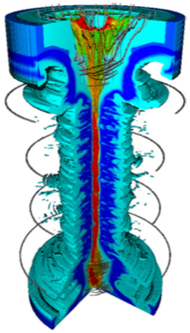
Time-integrated self-emission (FWHM~50-100 μm)

T.J. Awe et al., PRL 111, 235005 (2013)

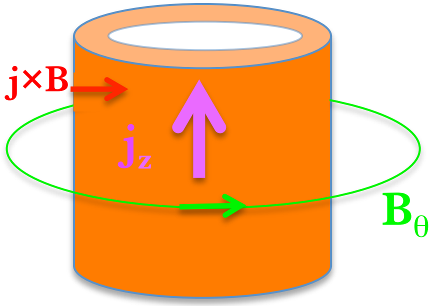
M.R. Gomez et al., PoP 22, 056306 (2015)



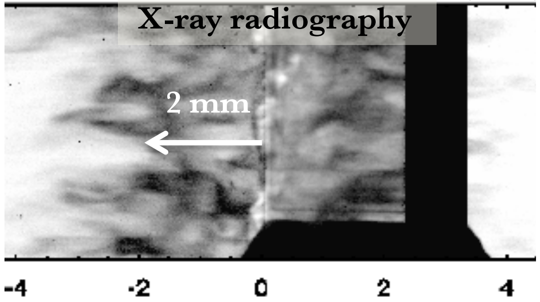
C. Jennings



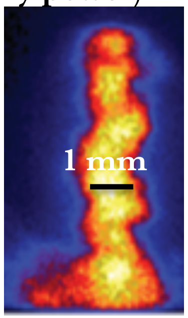
Cylindrical: wire array Z pinch (radiation source)

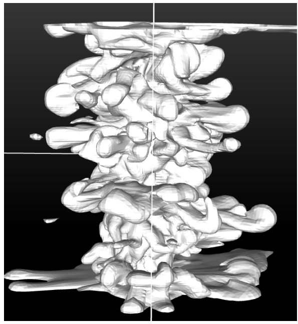


M.E. Cuneo et al., PoP **13**, 056318 (2006)



self-emission (peak x-ray power)



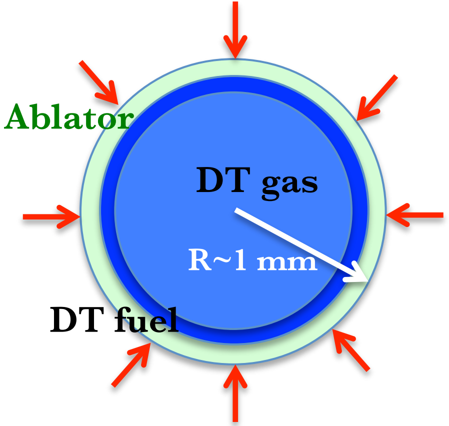


D.B. Sinars et al., PRL 100**,** 145002 (2008)

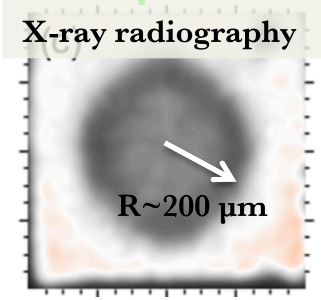
Like ICF capsules/MagLIF, the Z pinch is 3D

3D Simulation

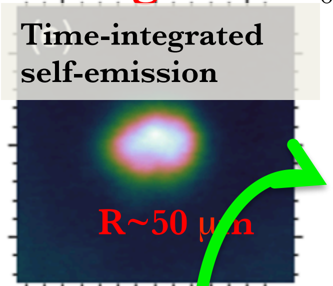
Spherical: ICF capsule



implosion



stagnation



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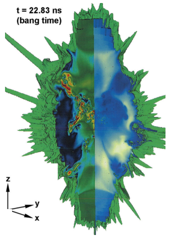

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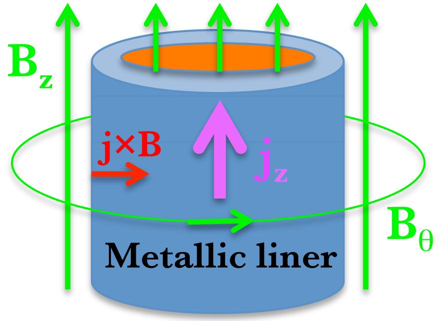
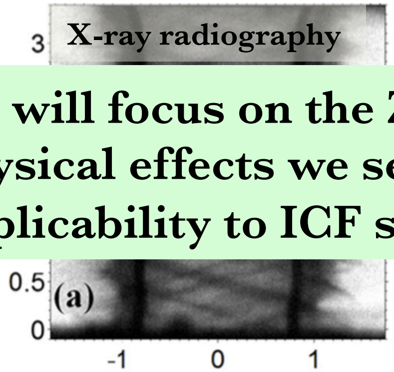
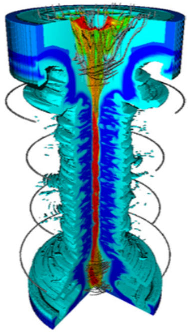




Time-integrated self-emission

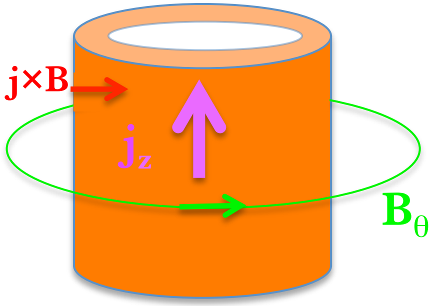
C. Jennings



We will focus on the Z pinch, but the physical effects we see here may have applicability to ICF systems.

M.R. Gomez et al., PoP 22, 056306 (2015)

Cylindrical: wire array Z pinch (radiation source)



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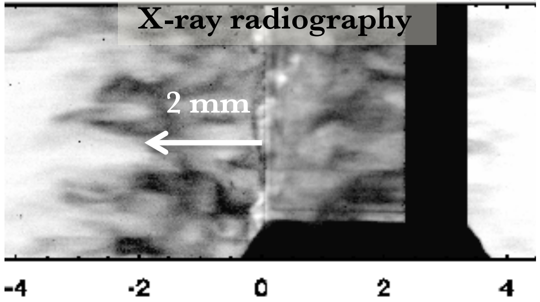

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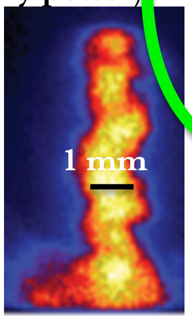
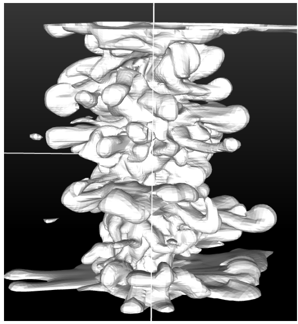




D.B. Sinars et al., PRL 100**,** 145002 (2008)

3D wire-array Z pinch simulation

Alegra: 3D radiation MHD + thermal conduction

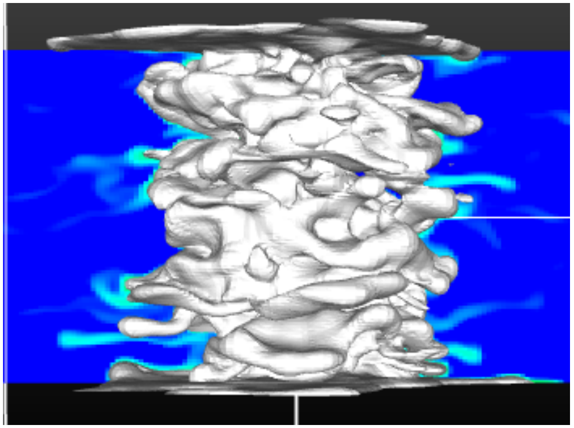
Eulerian mesh

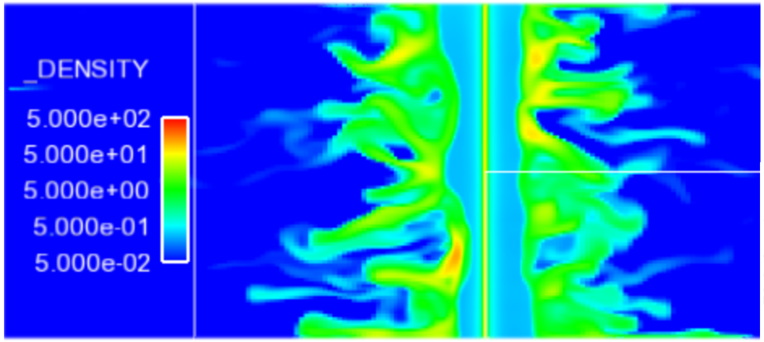
Artificial viscosity

1.7 million elements (dz~60 μm, dr~20 μm)

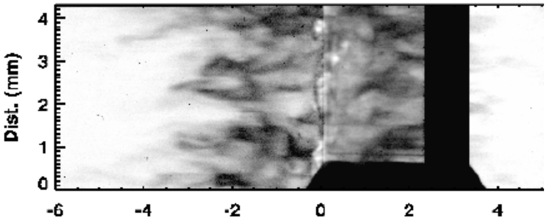
Mass injection scheme*

ρ=1 kg/m^3 surface

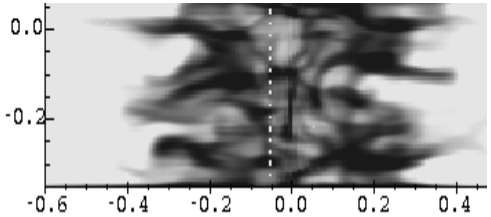


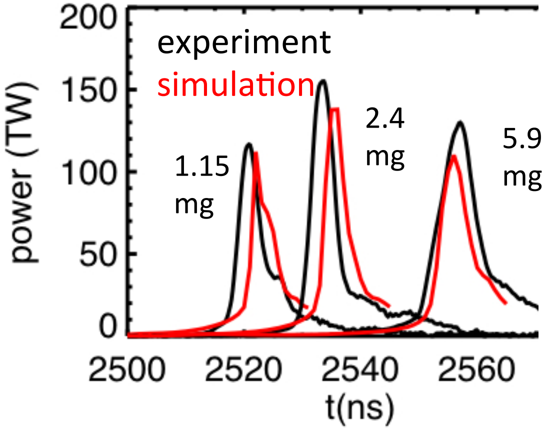


Experimental radiograph



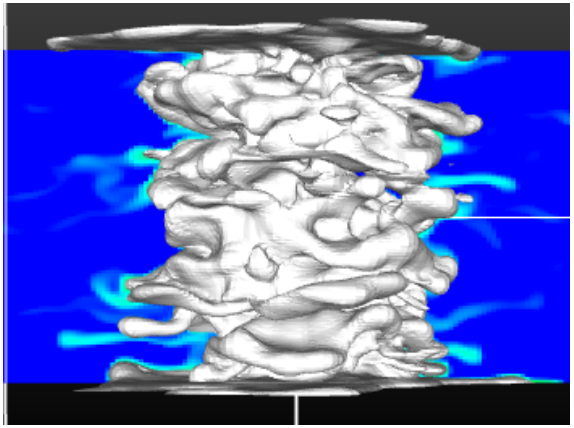
Simulation radiograph





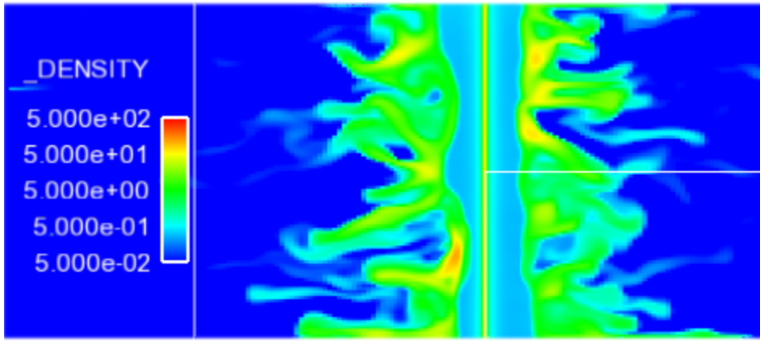
*E.P. Yu, M.E. Cuneo, M. P. Desjarlais, R.W. Lemke, et al., Phys. Plasmas 15, 056301 (2008)

3D wire-array Z pinch simulation



t=-2.4 ns

ρ=1 kg/m^3 surface



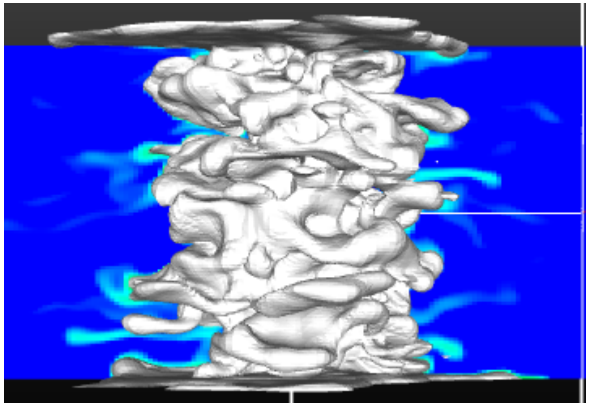
Simulate 1.15 mg, tungsten compact array (R_o=1cm)

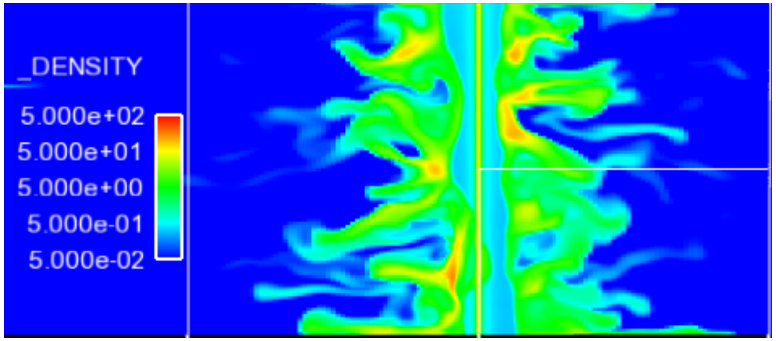
At t=-2.4 ns ns, radiation is "turned off" (σ_r/1e4)

3D wire-array Z pinch simulation

t=-2 ns

ρ=1 kg/m^3
surface





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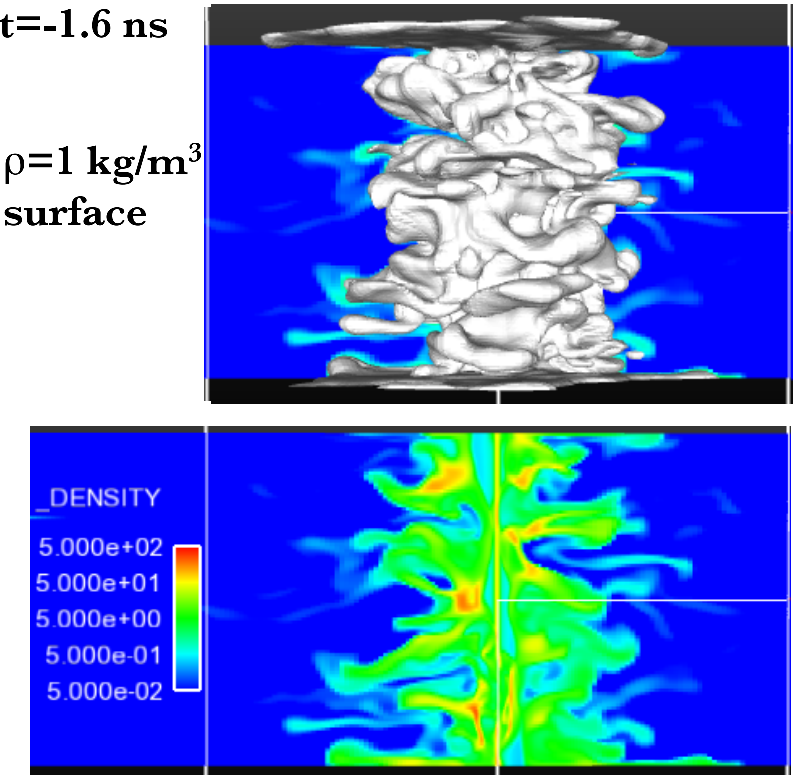
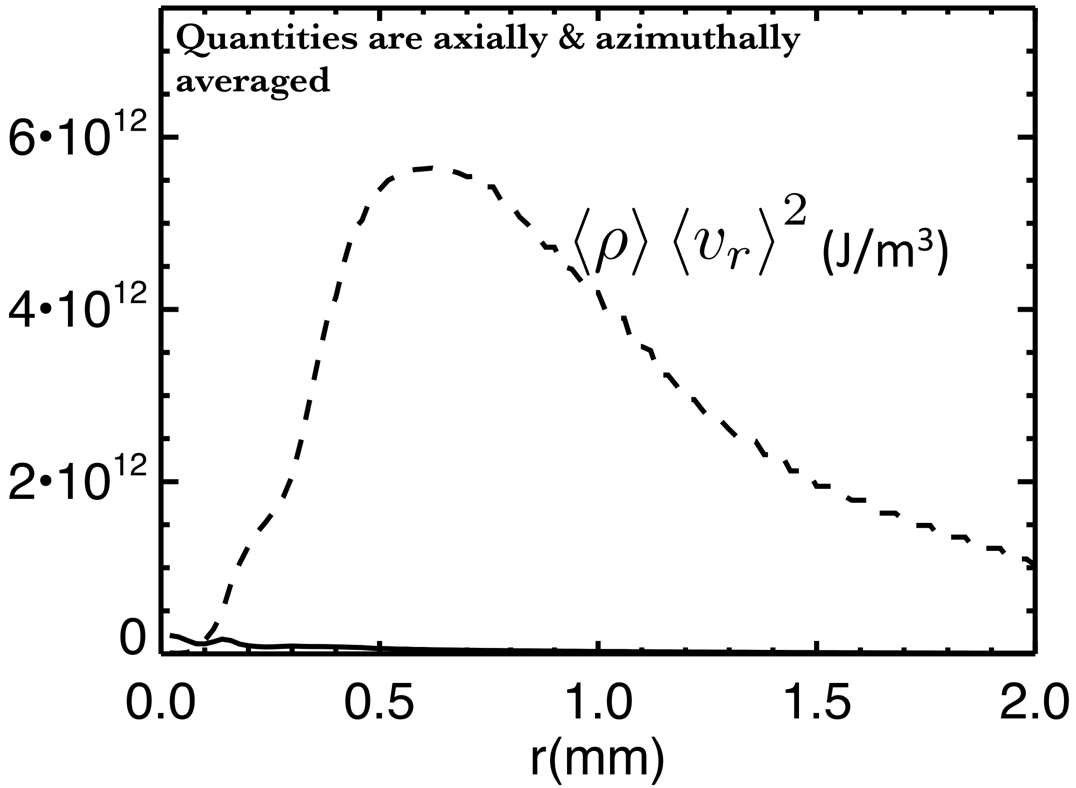
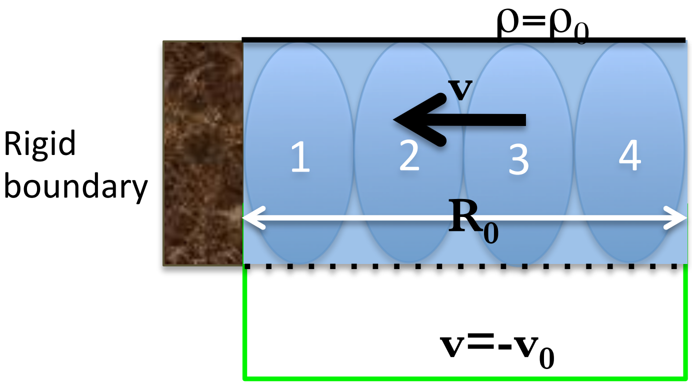




What's the significance of this non-monotonic ram pressure?

1D shock solution

1D planar geometry



Consider cold fluid particles imploding towards a rigid boundary. Density and velocity profiles are flat.

Problem we will consider is purely hydrodynamic.

This is the “Noh problem”*

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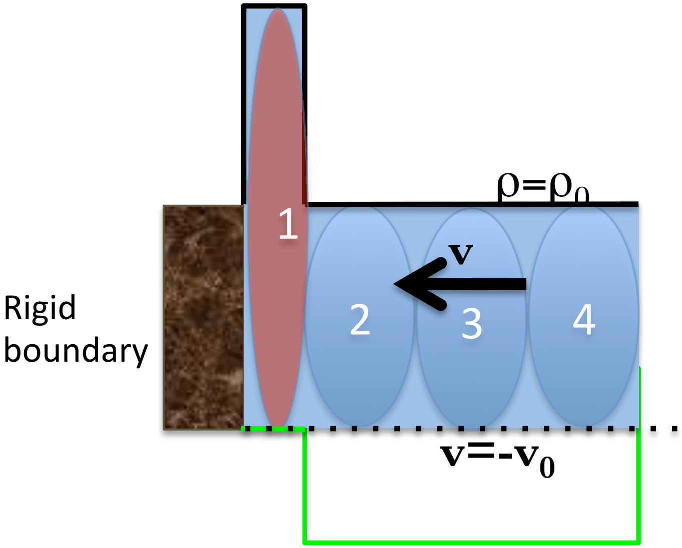
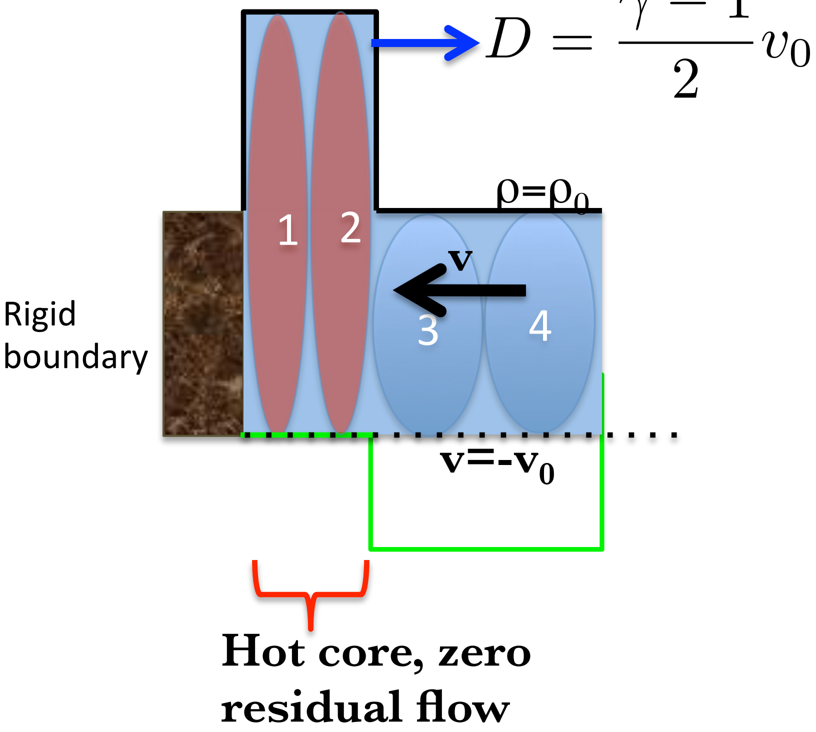
*W.F. Noh, J. Comp. Phys. 72, 78 (1987)

1D shock solution



Particle 1 collides into the boundary and converts all its kinetic energy into internal energy.

1D shock solution

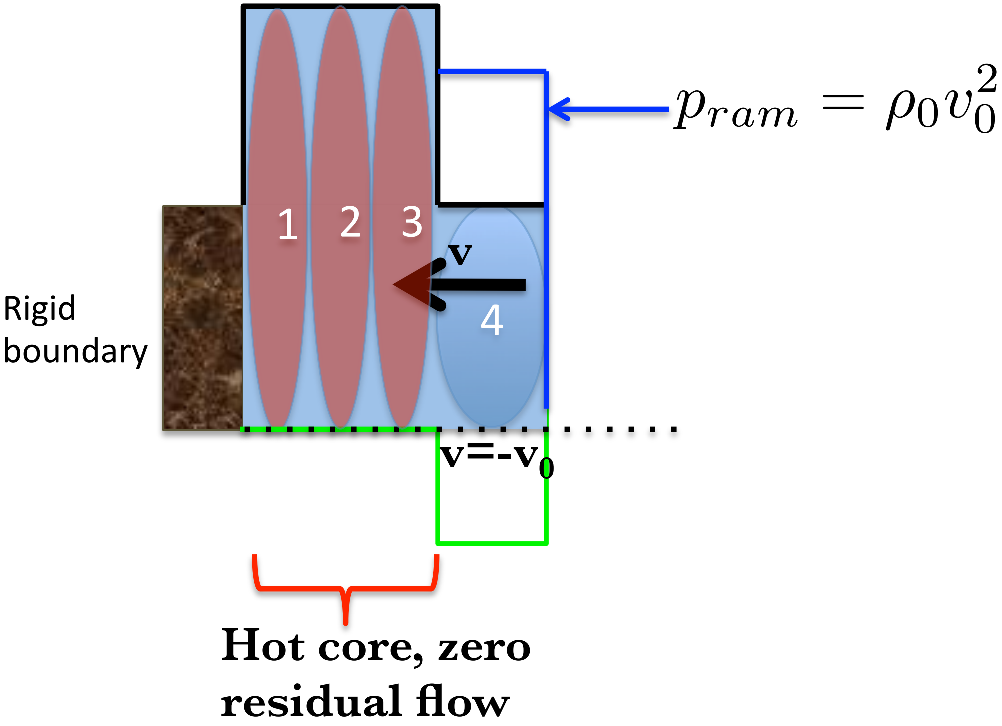


$$D = \frac{\gamma - 1}{2} v_0$$

Hot core, zero residual flow

Hot core grows outward through *shock accretion*.

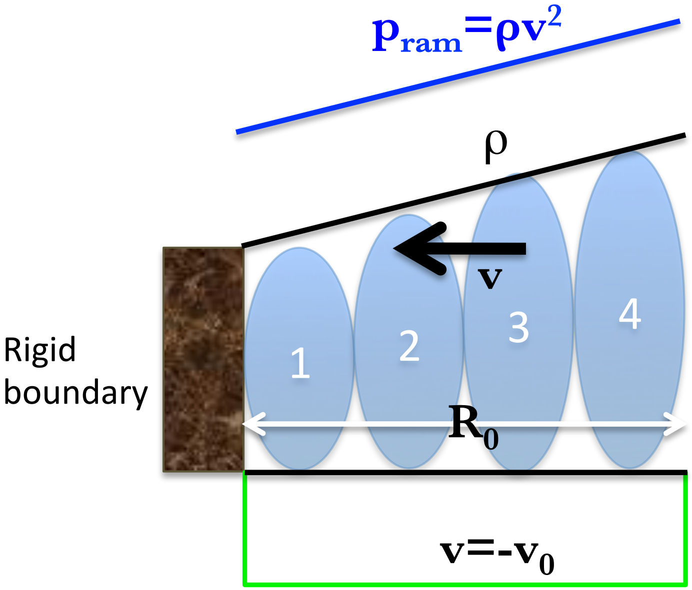
1D shock solution



The hot core is confined in the sense that v=0 there.

This confinement is due solely to the incoming ram pressure.

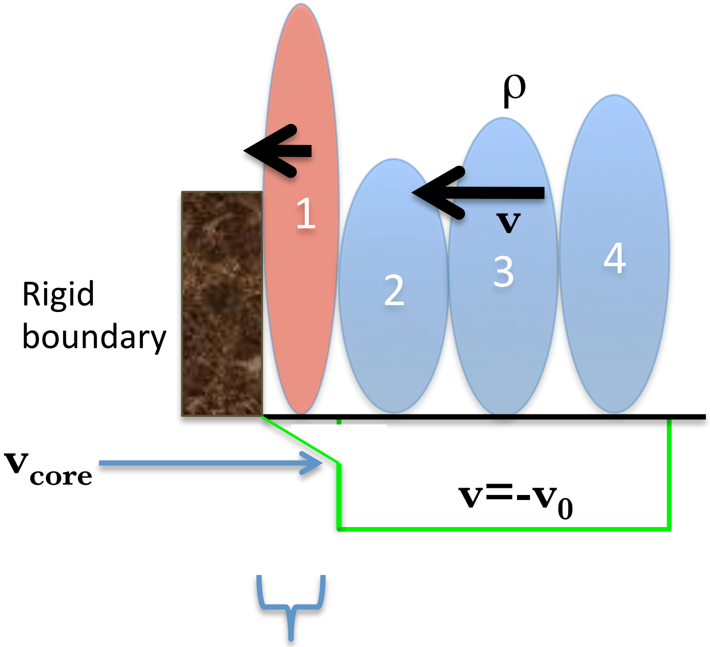
1D shock solution: increasing p_{ram}



The Noh problem has been generalized by A. Velikovich to allow for spatially varying ρ(r), v(r).

Density and ram pressure profiles are increasing.

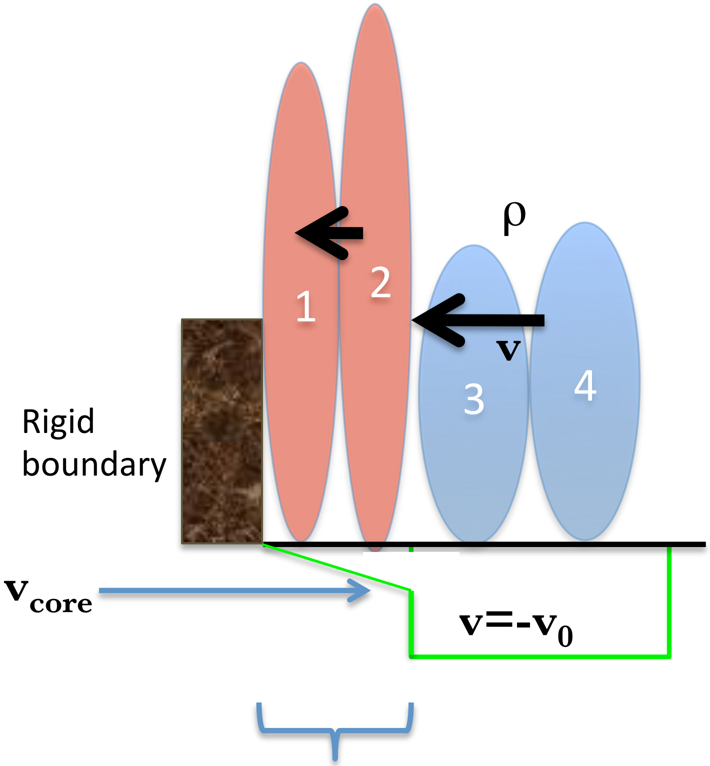
1D shock solution: increasing p_{ram}



Residual flow in core

Because of increasing ram pressure, shocked fluid particle continues to compress.

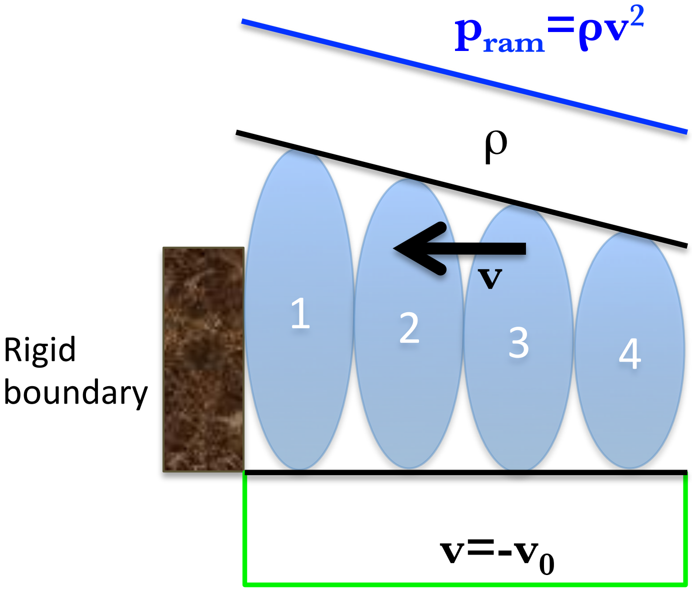
1D shock solution: increasing p_{ram}



Residual flow in core

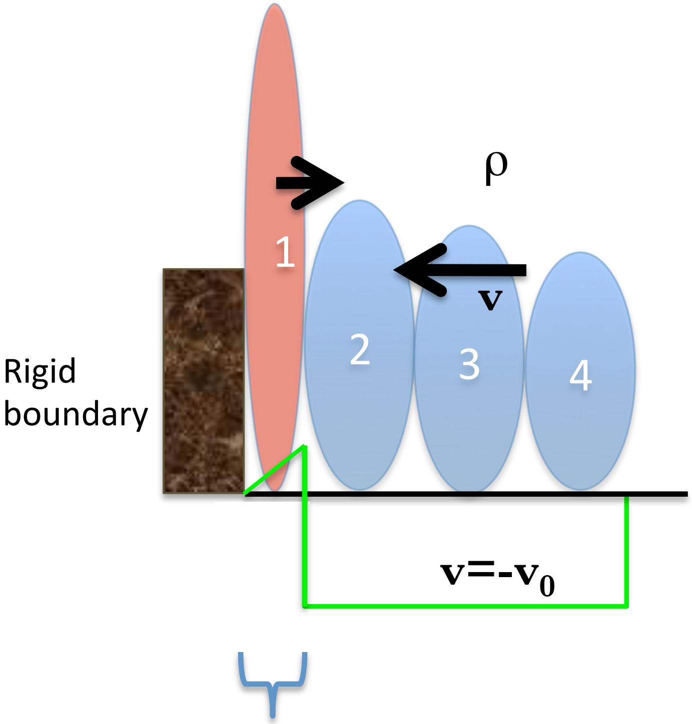
Because of continued compression in core, density there rises with time.

1D shock solution: decreasing p_{ram}



Cold fluid particles implode towards a rigid boundary. Density and ram pressure profiles are decreasing.

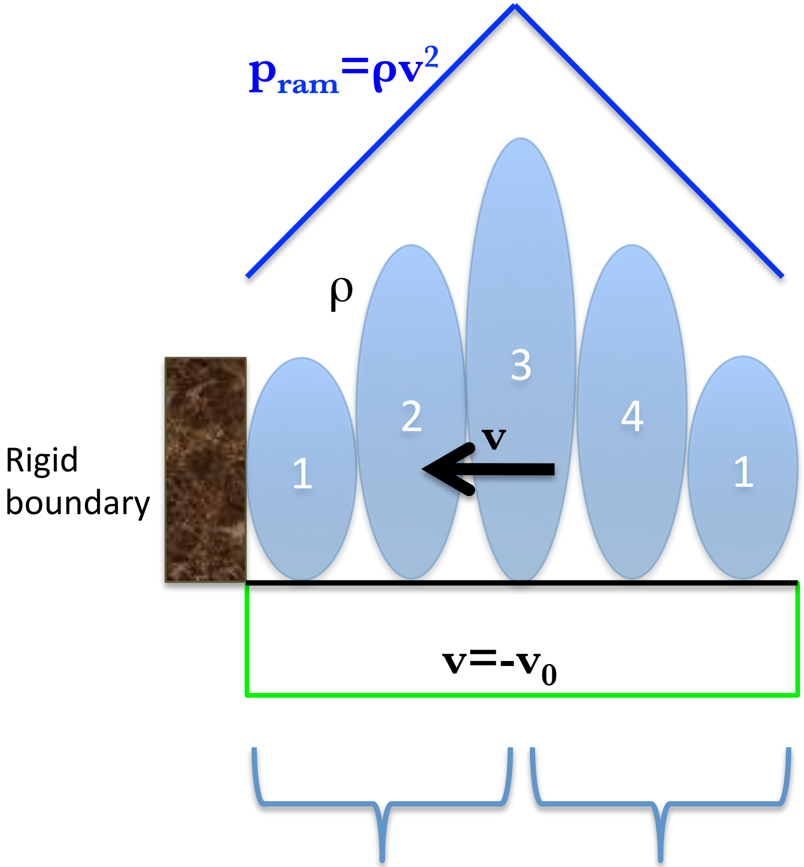
1D shock solution: decreasing p_{ram}



Residual flow in core

Because of decreasing ram pressure, shocked fluid particle expands outward into imploding plasma.

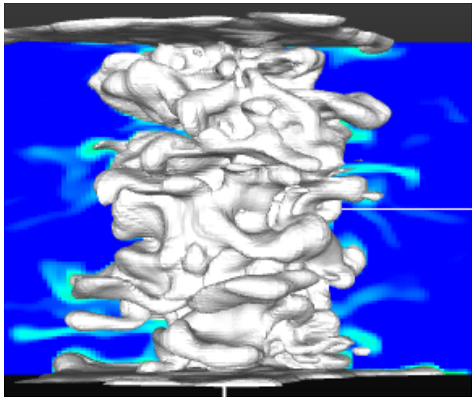
1D shock solution: non-monotonic p_{ram}

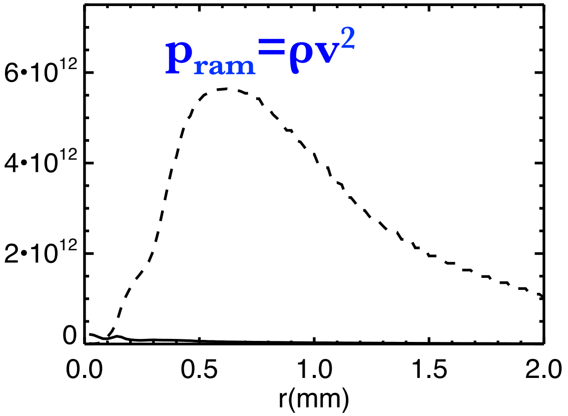


During rising portion of p_{ram}, core compresses

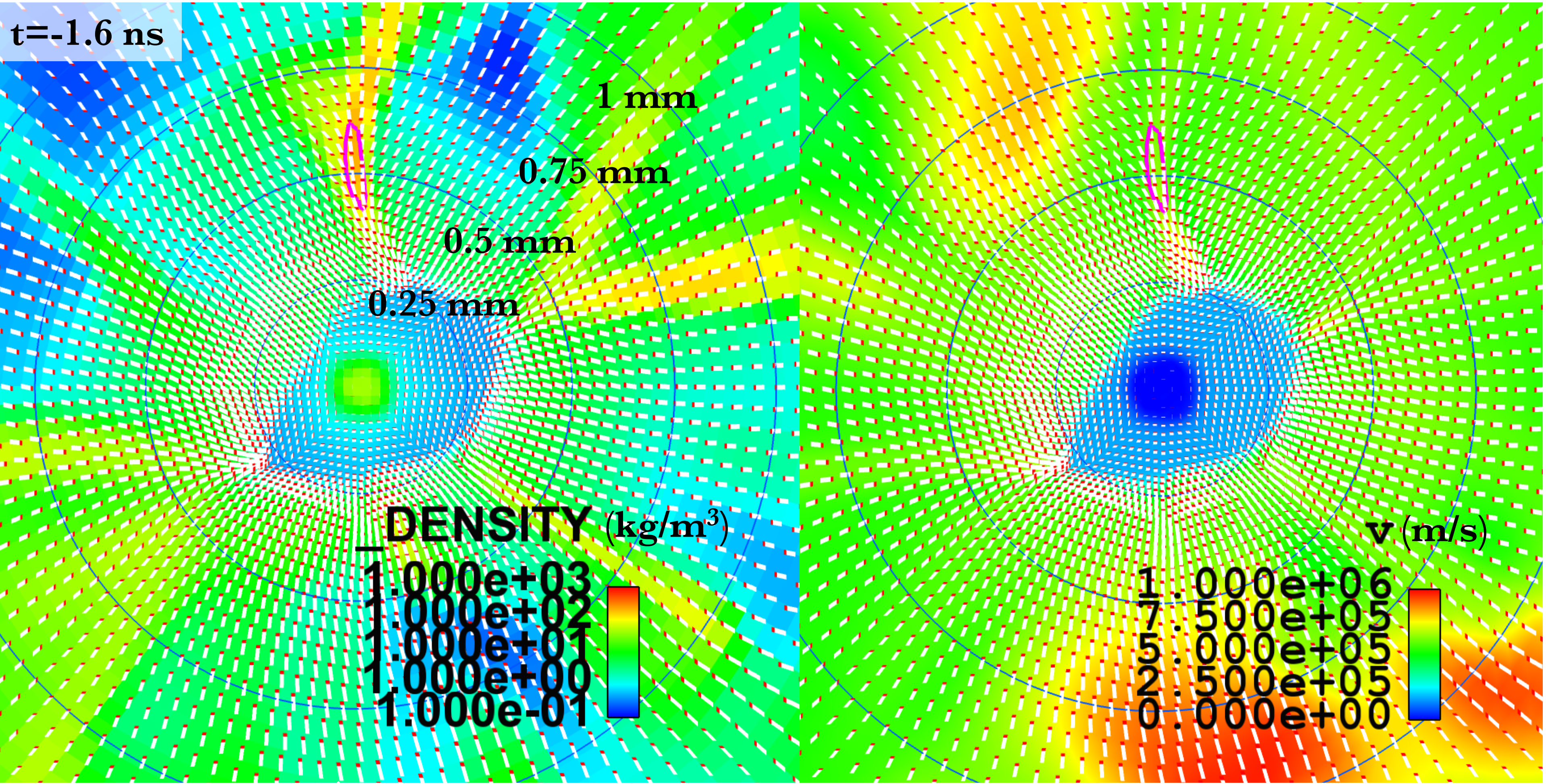
During falling portion of p_{ram}, core expands

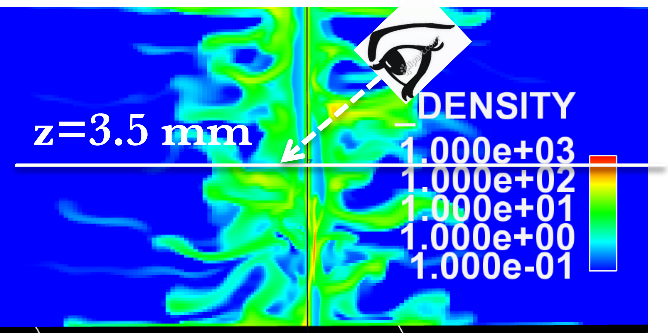
3D simulation also shows non-monotonic p_{ram}

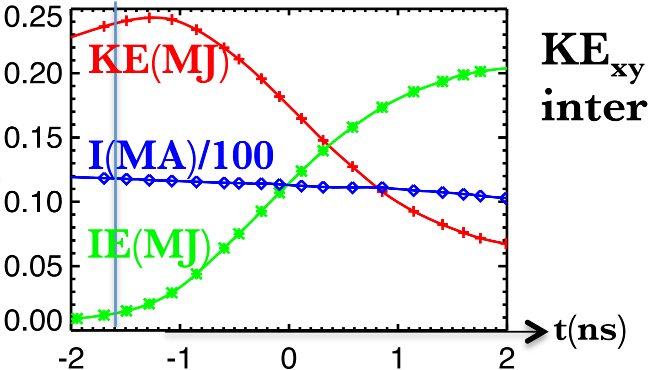




3D simulation fluid flows

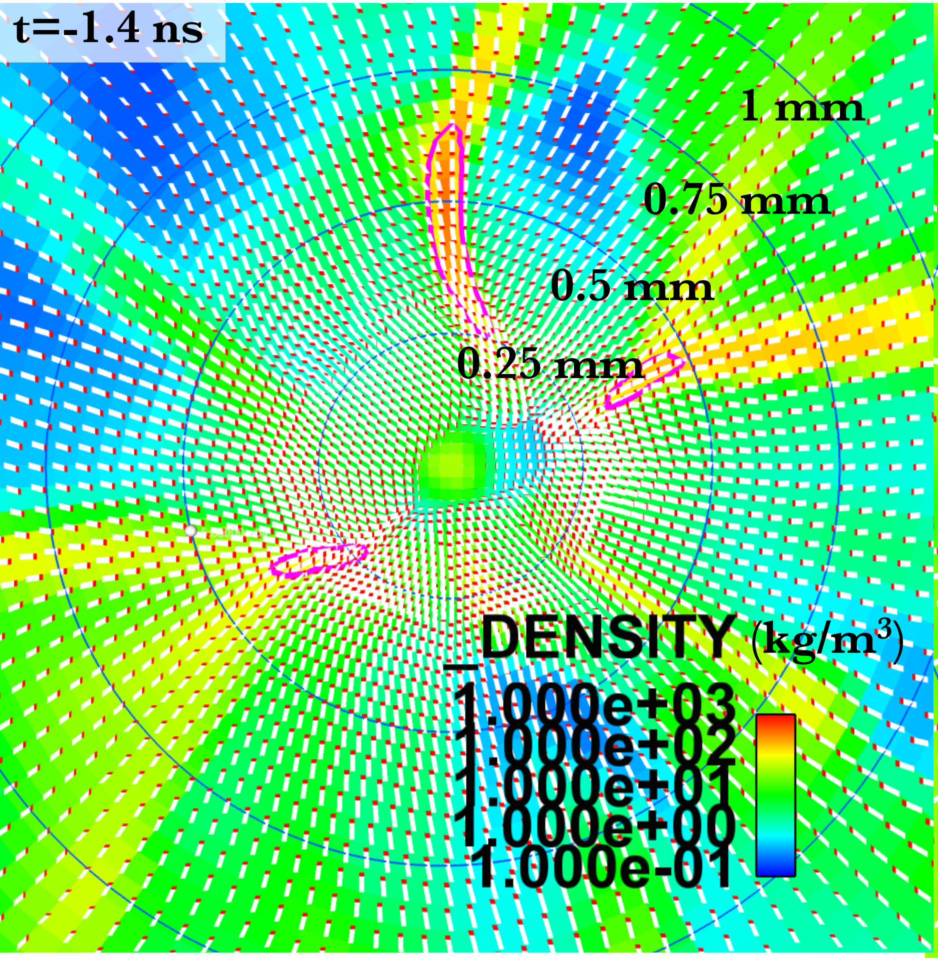


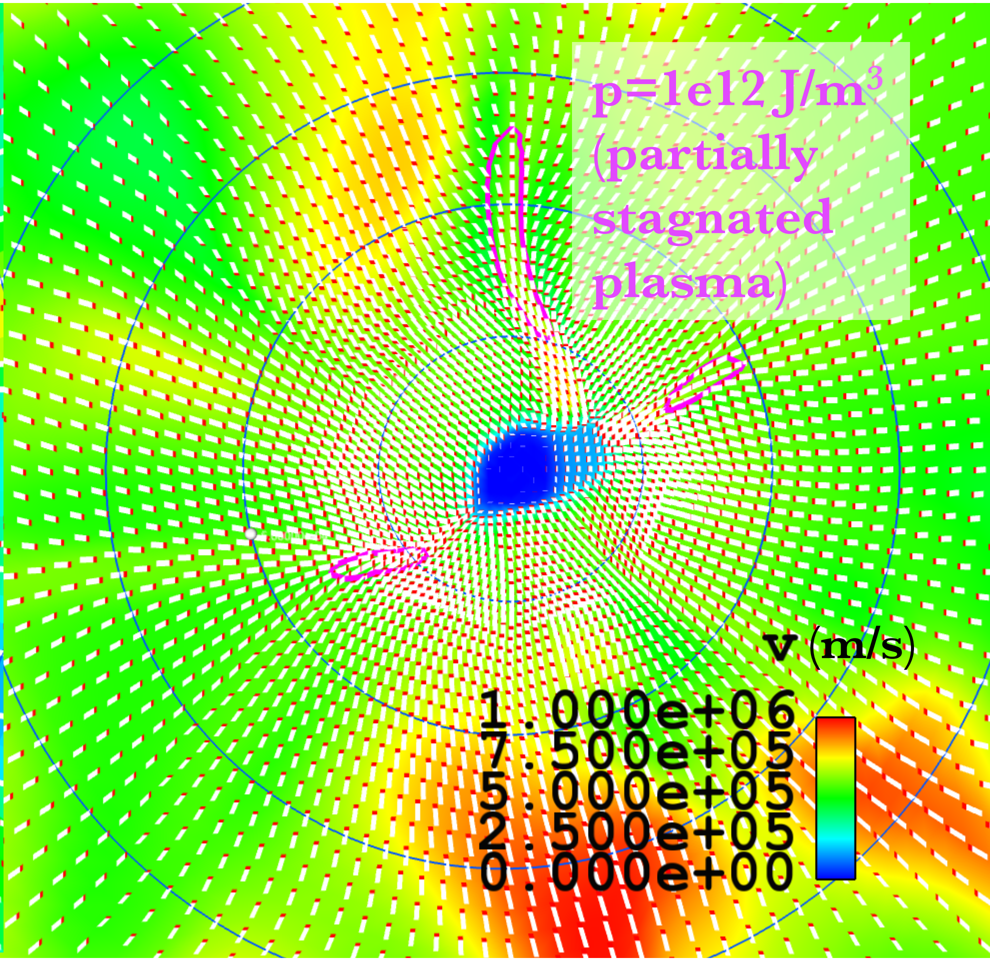


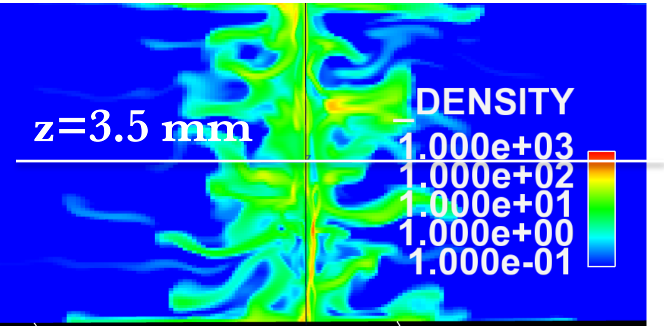


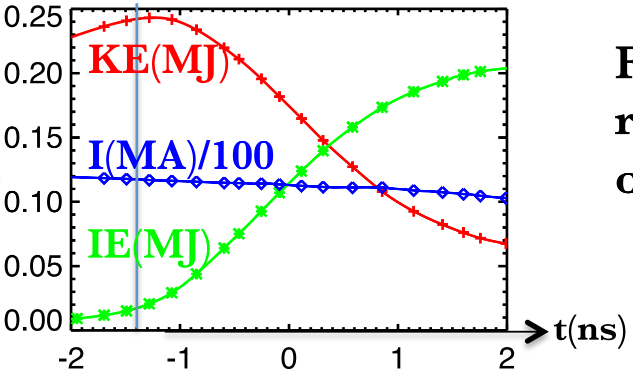
KE_{xy}~4*KE_z during times of interest

3D simulation fluid flows



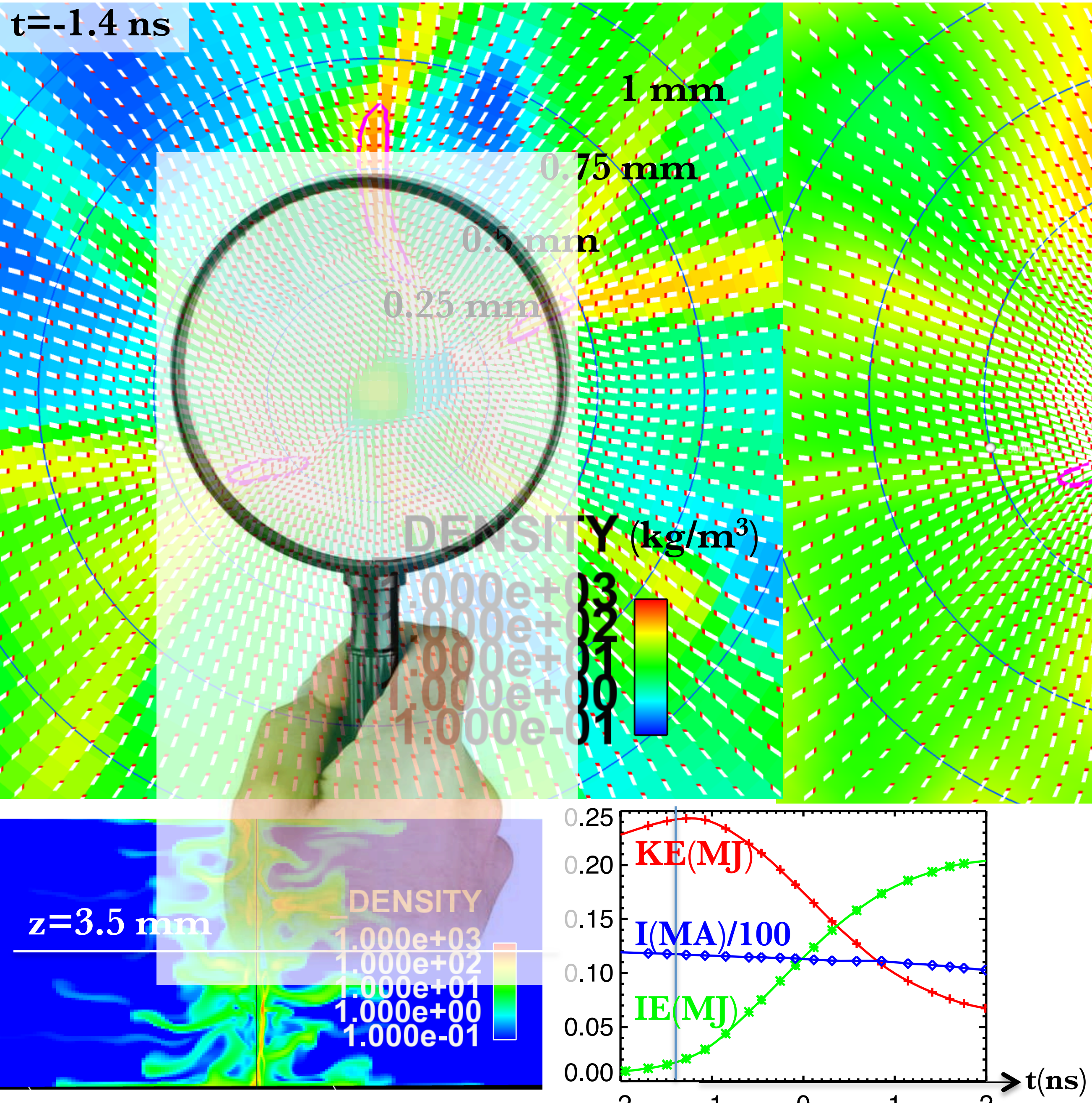




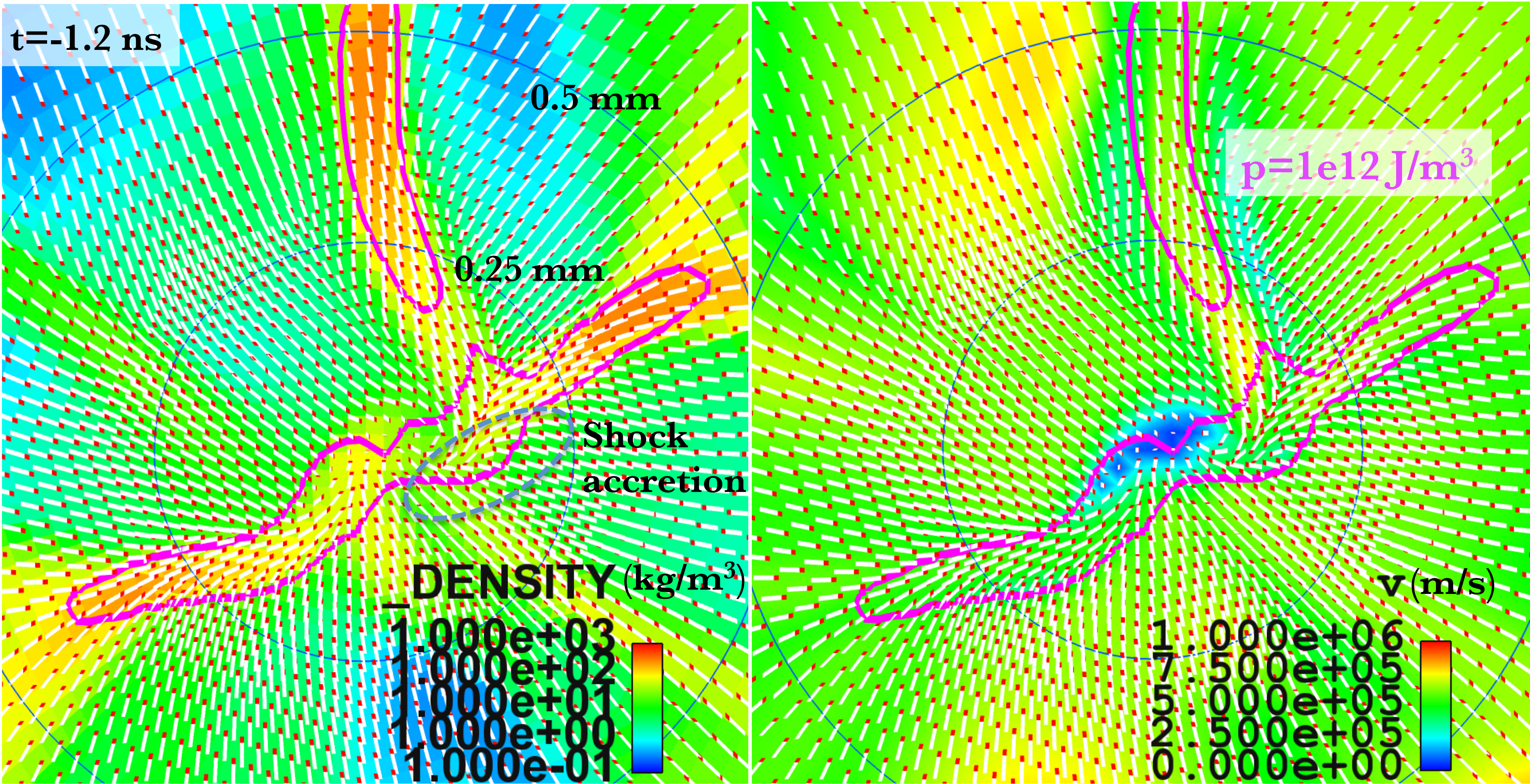


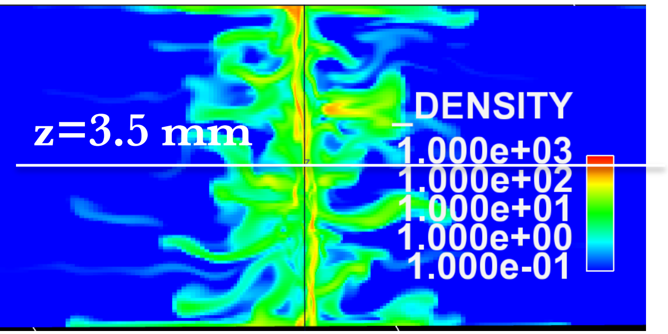
Flow is not completely radial; plasma collides obliquely and off-axis

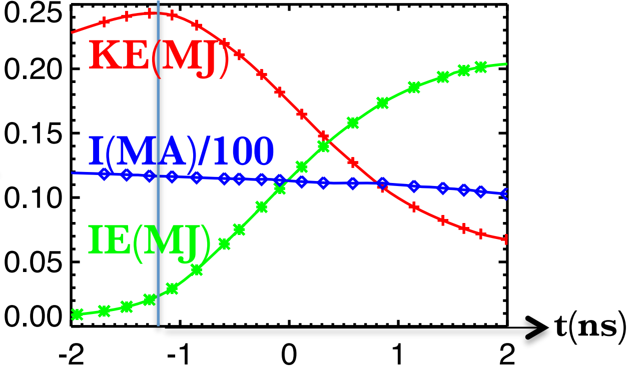
3D simulation fluid flows



3D simulation fluid flows







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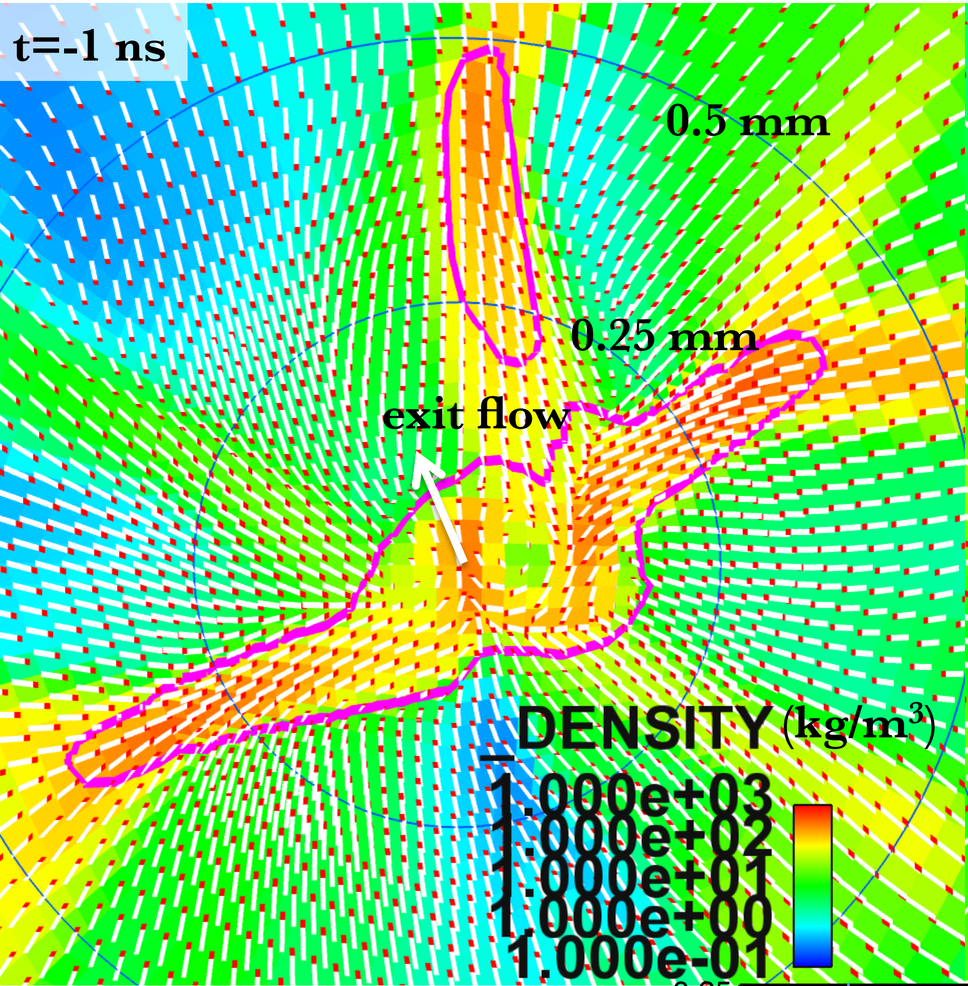
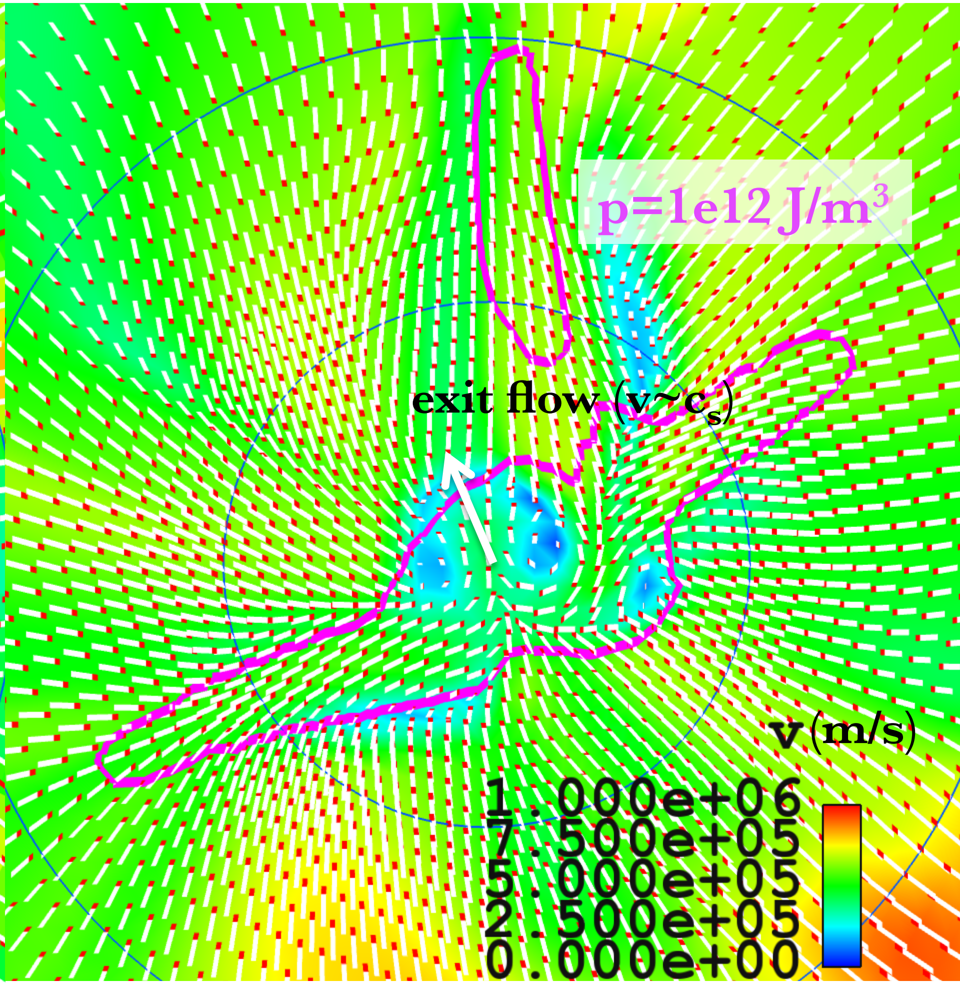
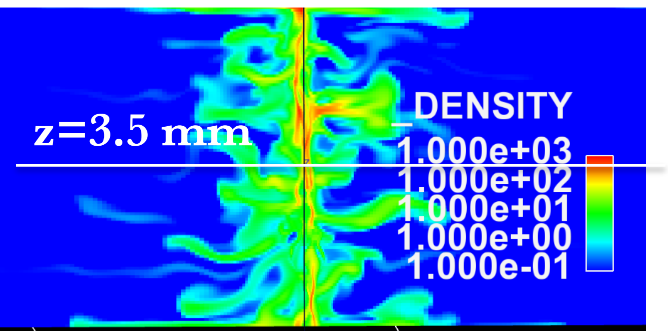
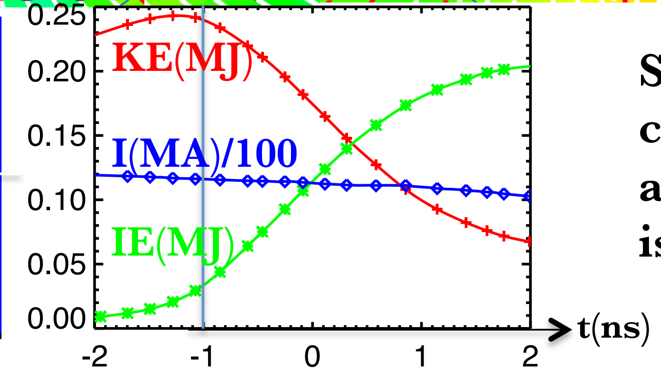
3D simulation fluid flows



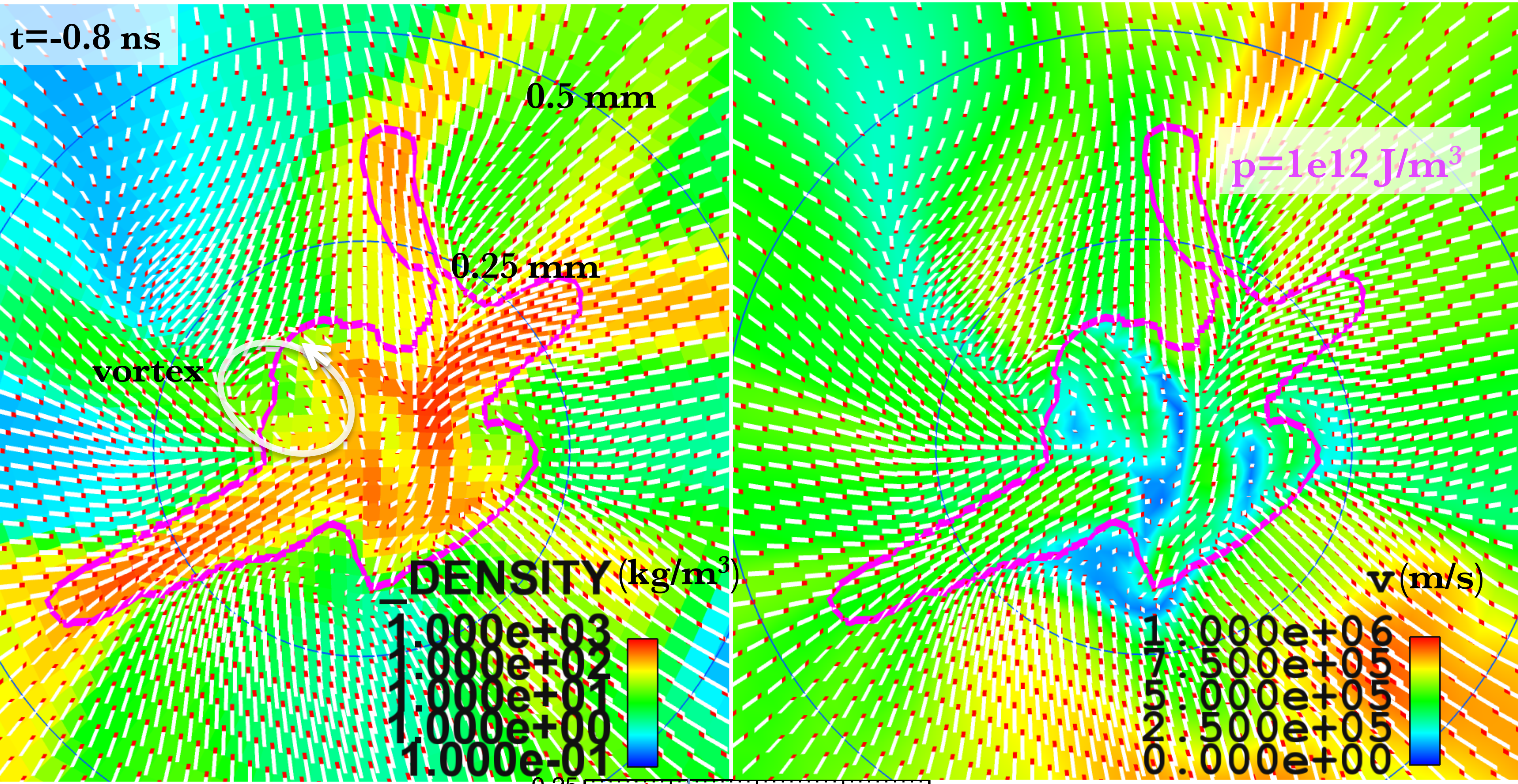


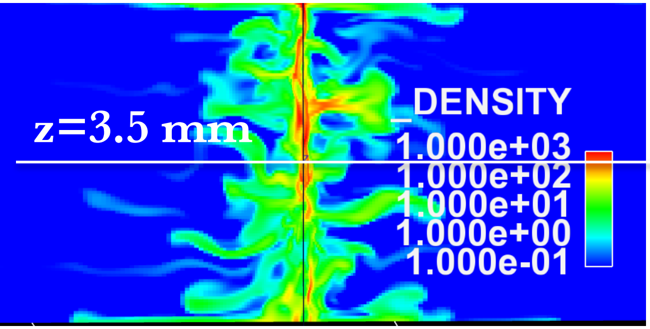


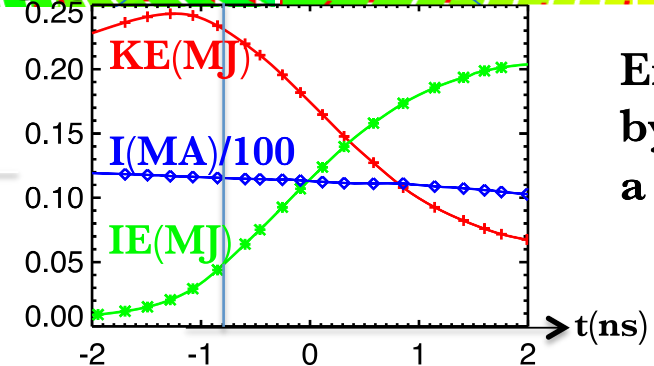


Stagnated region (purple contour) is growing via accretion. "Fast" exit flow is forming

3D simulation fluid flows

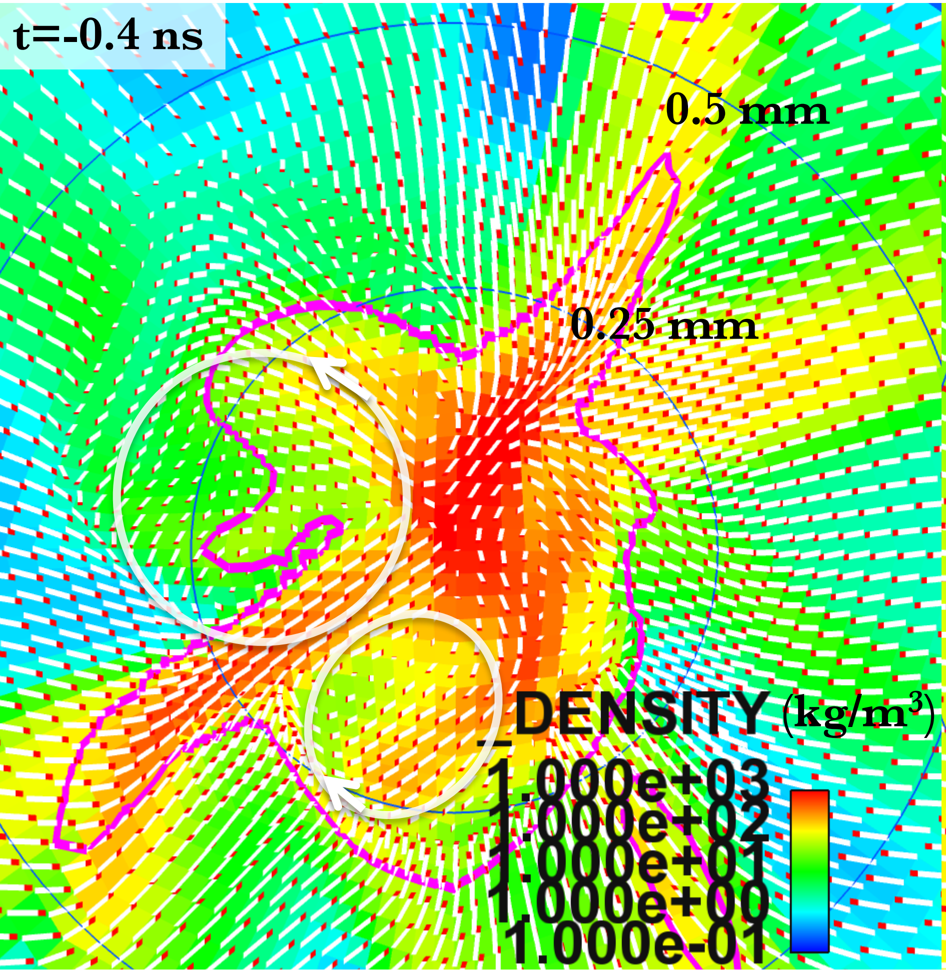


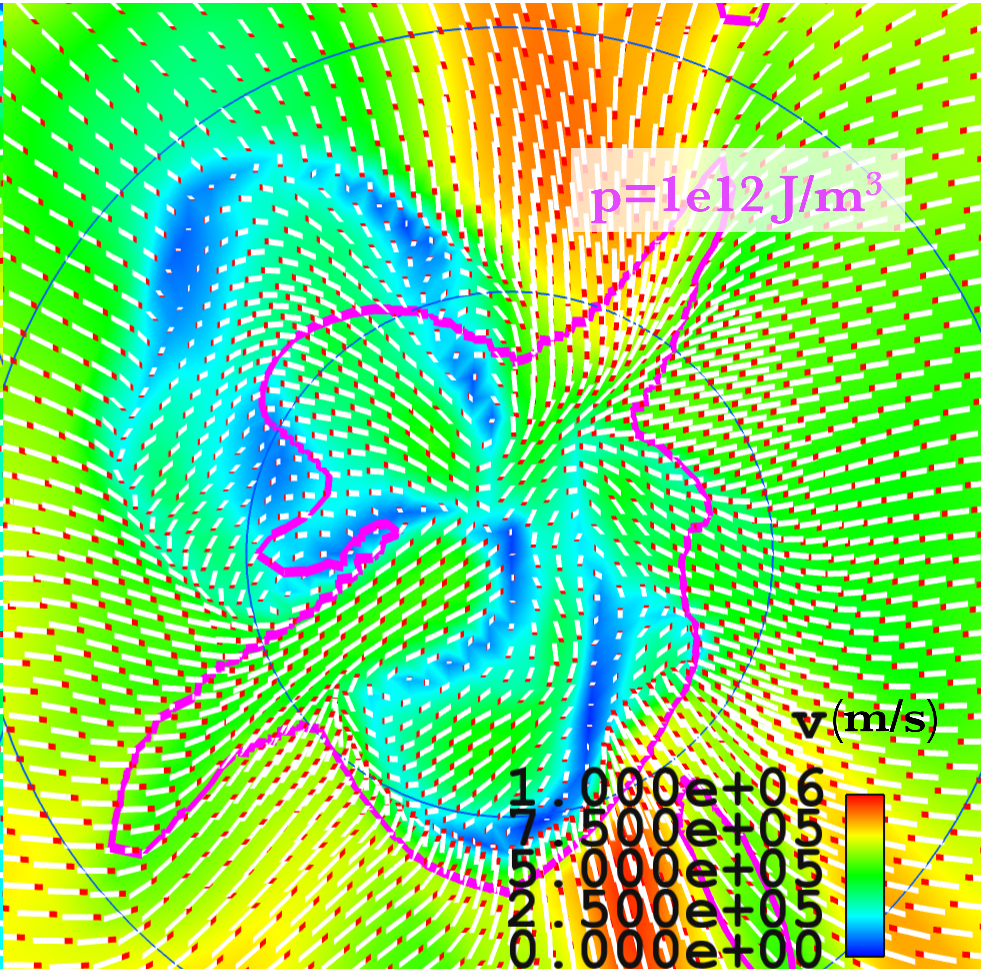


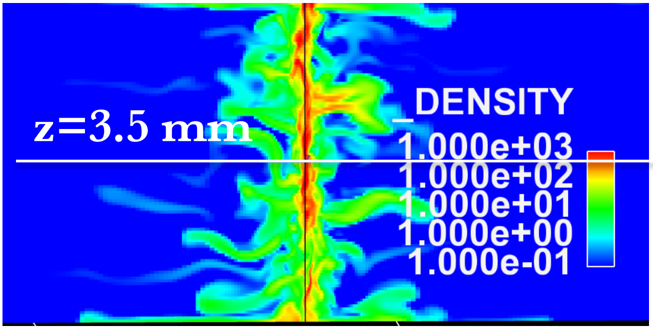


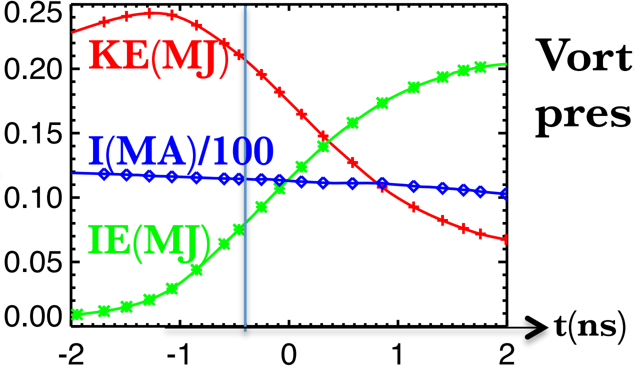
Exit flow is turned around by ram pressure, forming a vortex

3D simulation fluid flows









Vortices generate centrifugal pressure

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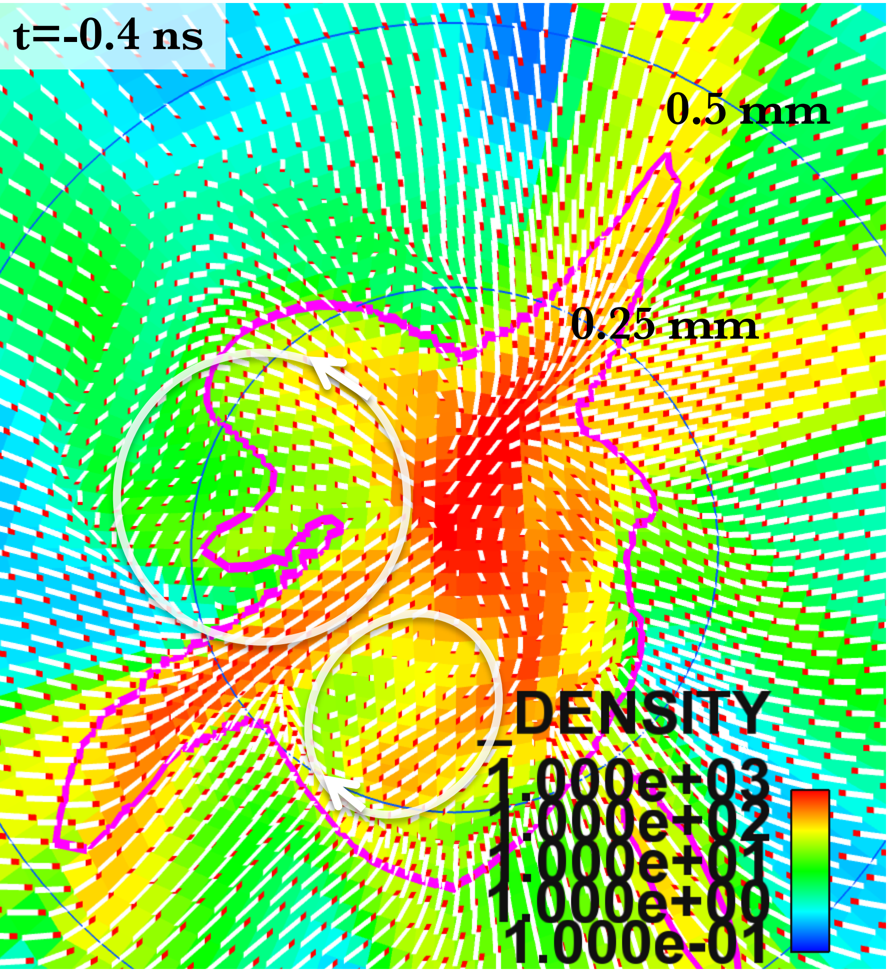
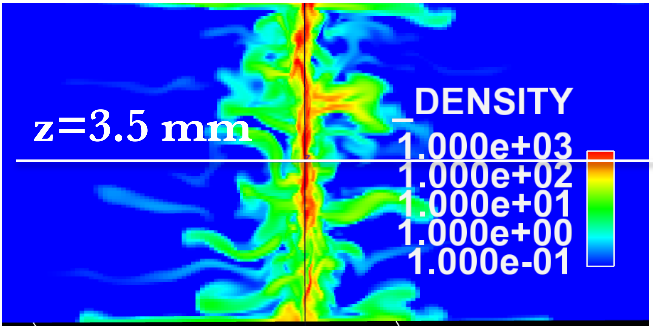
Effect of residual flows on pressure balance





How much "pressure" do the vortices contribute?

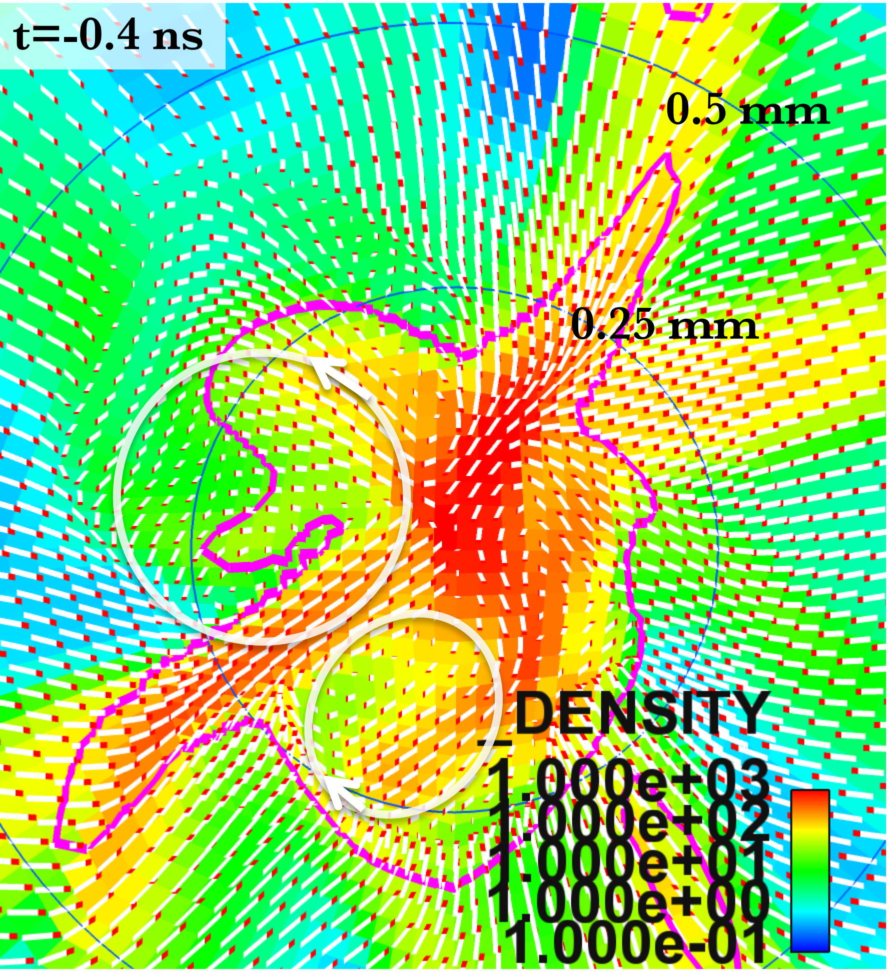
$$\rho\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right)=-\nabla p+\mathbf{J}\times\mathbf{B}$$

radial component:

$$\rho\frac{\partial v_r}{\partial t}=-\underbrace{\frac{\partial p}{\partial r}}_{\text{pressure gradient}}-\rho\left[v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}+v_z\frac{\partial v_r}{\partial z}-\underbrace{\frac{v_\theta^2}{r}}_{\text{centrifugal force}}\right]$$

$$+J_\theta B_z-J_zB_\theta$$

Centrifugal force is comparable to pressure gradient



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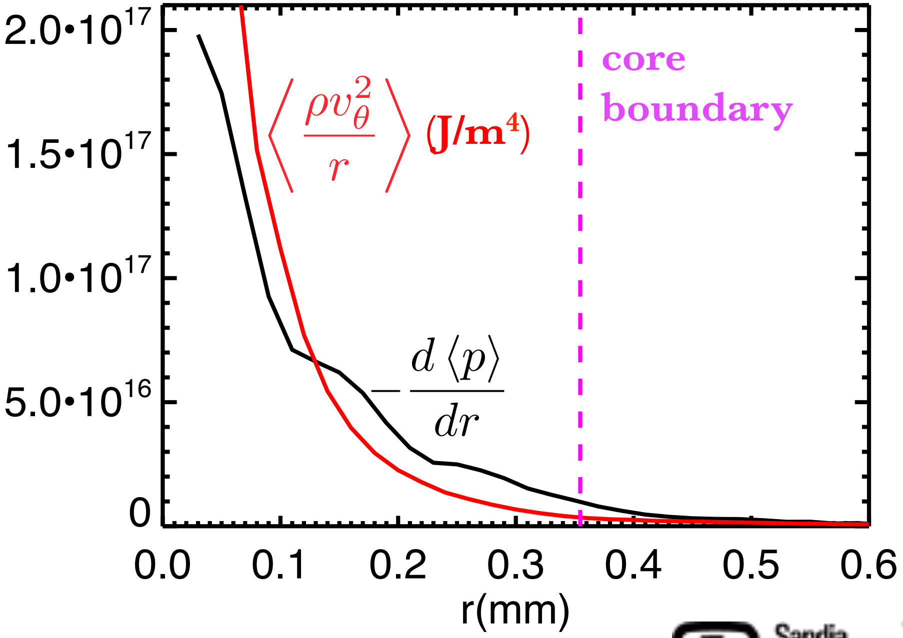
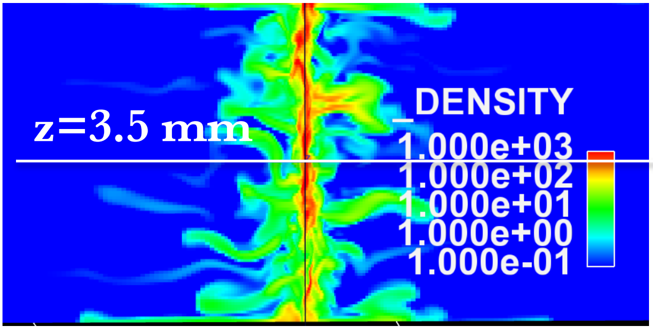
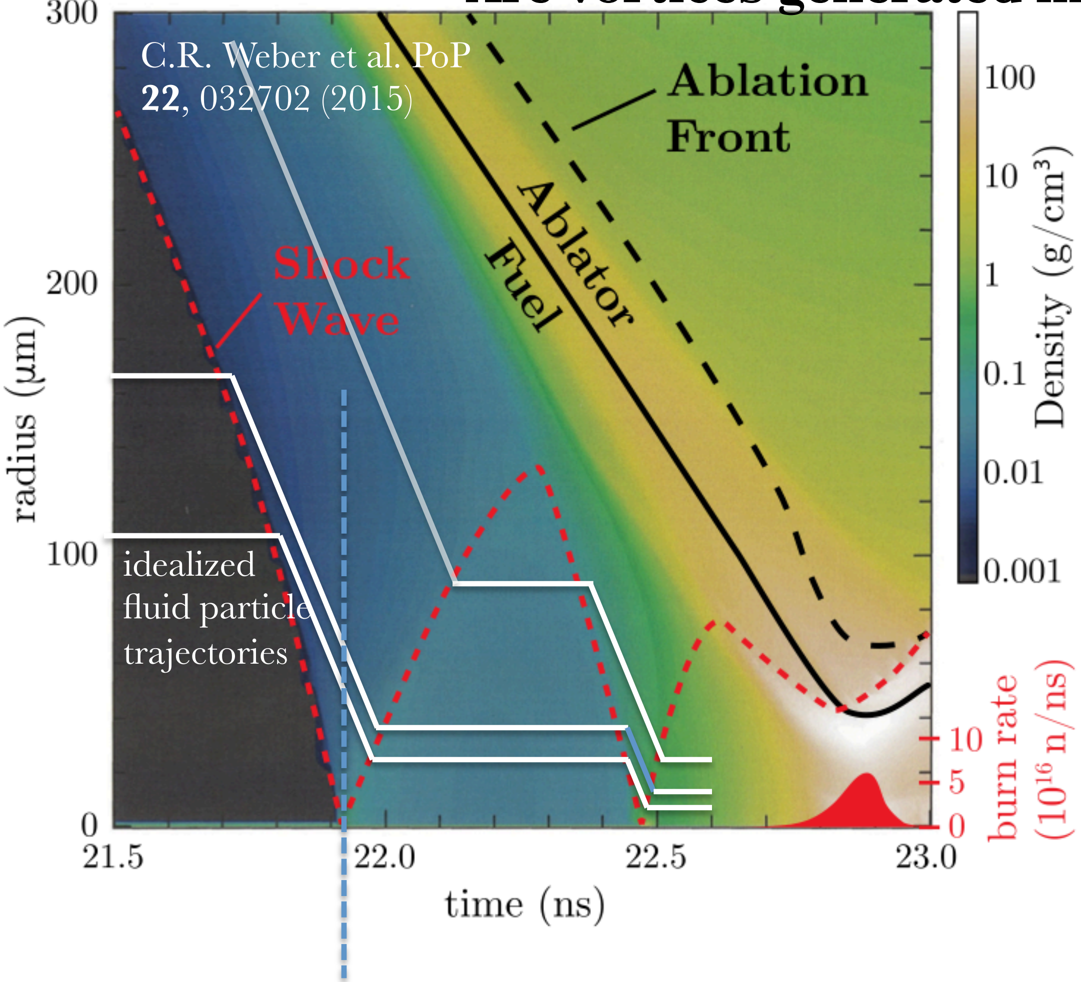


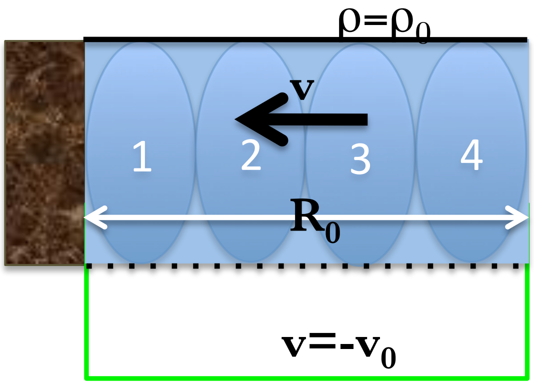


Y.M. Maron et al., PRL 111, 035001 (2013)

Are vortices generated in ICF capsules?


C.R. Weber et al. PoP **22**, 032702 (2015)

t_1 (transmitted shock about to hit axis)



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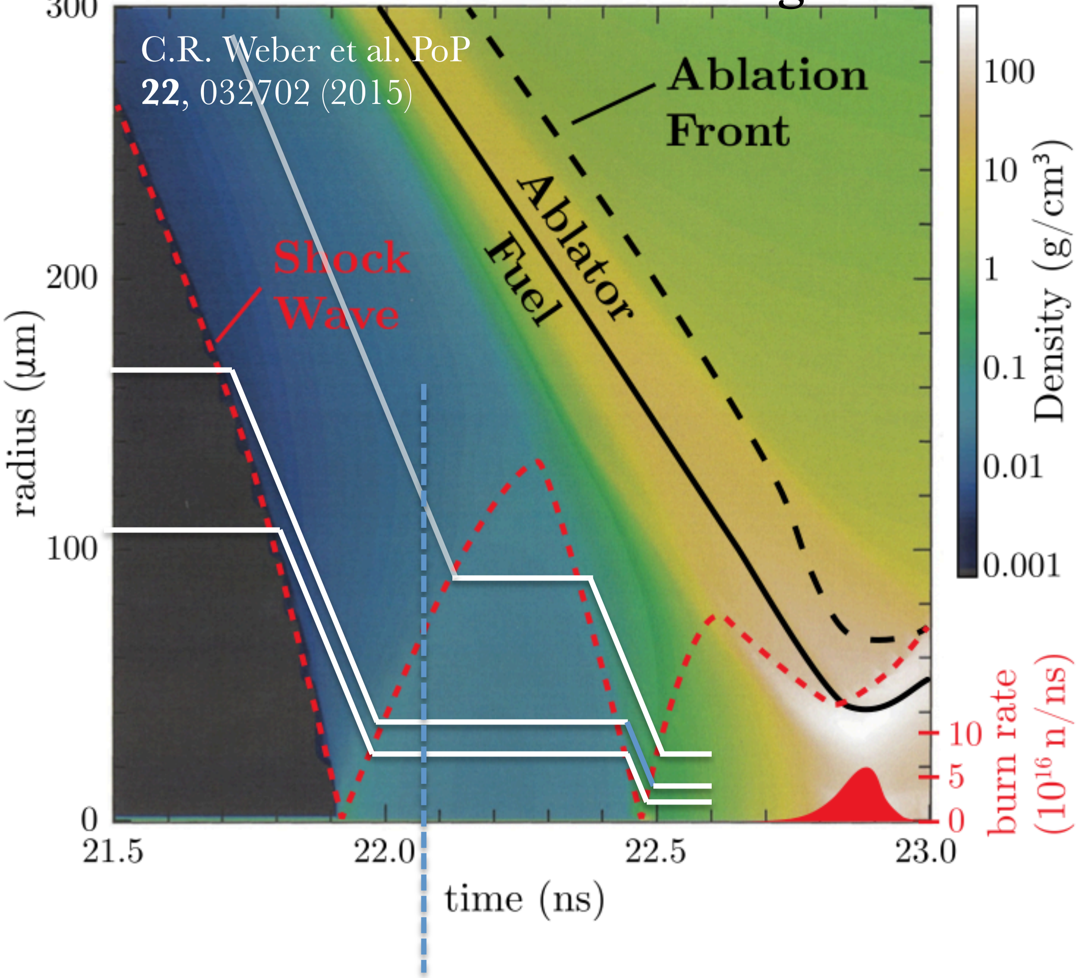
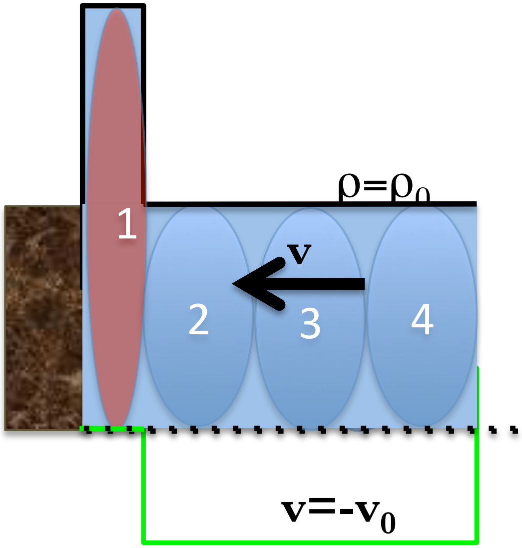
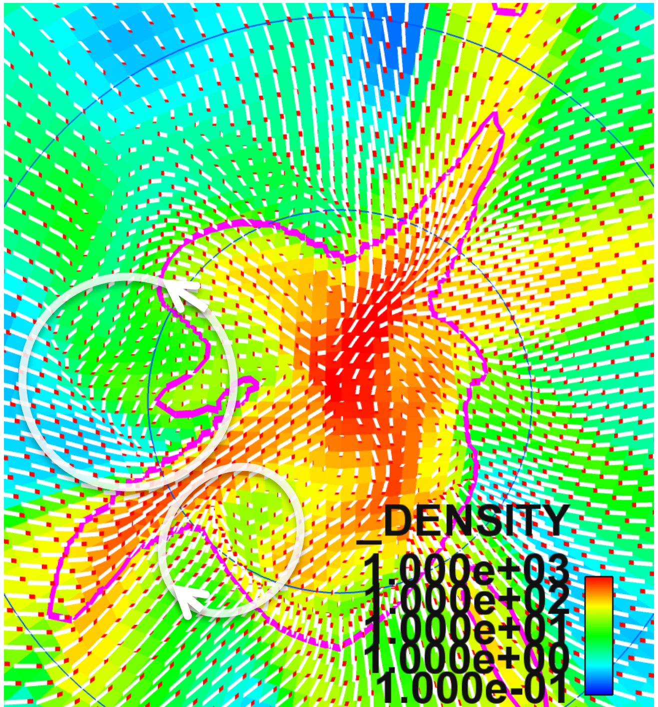


C.R. Weber et al. PoP **22**, 032702 (2015)

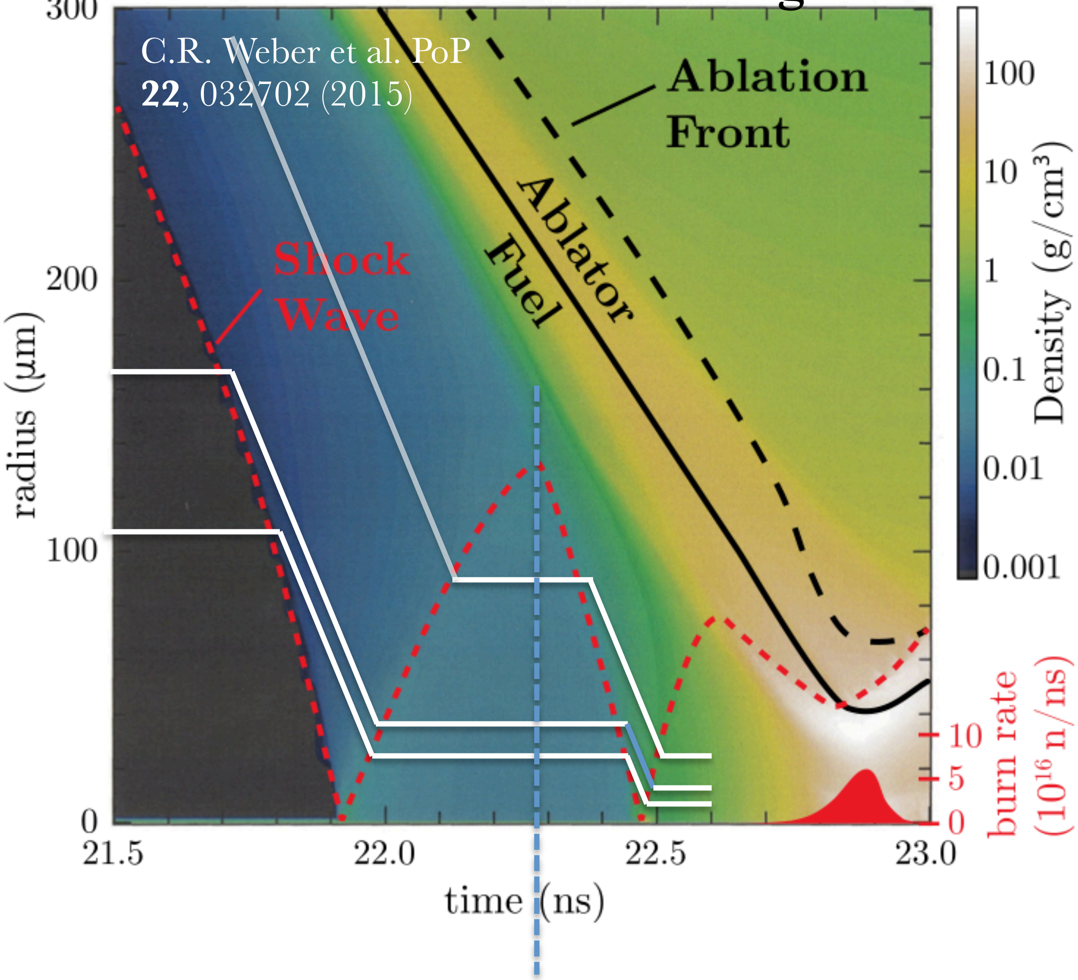
t_2 (shock reflection)



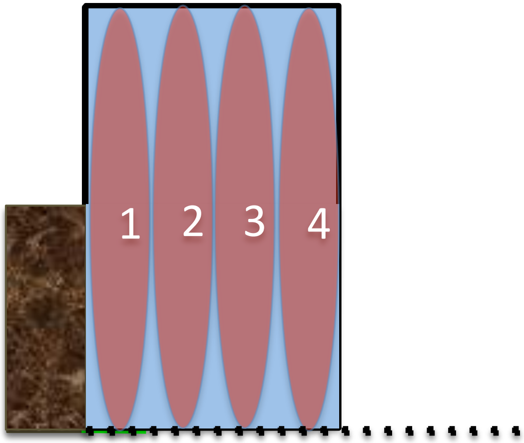


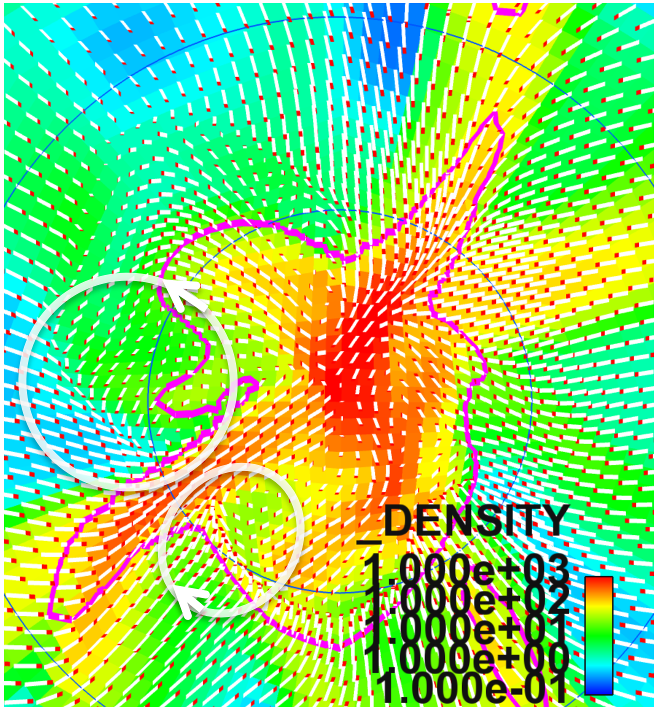
Possibly vortices are generated due to imbalance in pressure/ram pressure, as just described.

Are vortices generated in ICF capsules?


C.R. Weber et al. PoP **22**, 032702 (2015)

t$_3$(shock reflection in gas complete)





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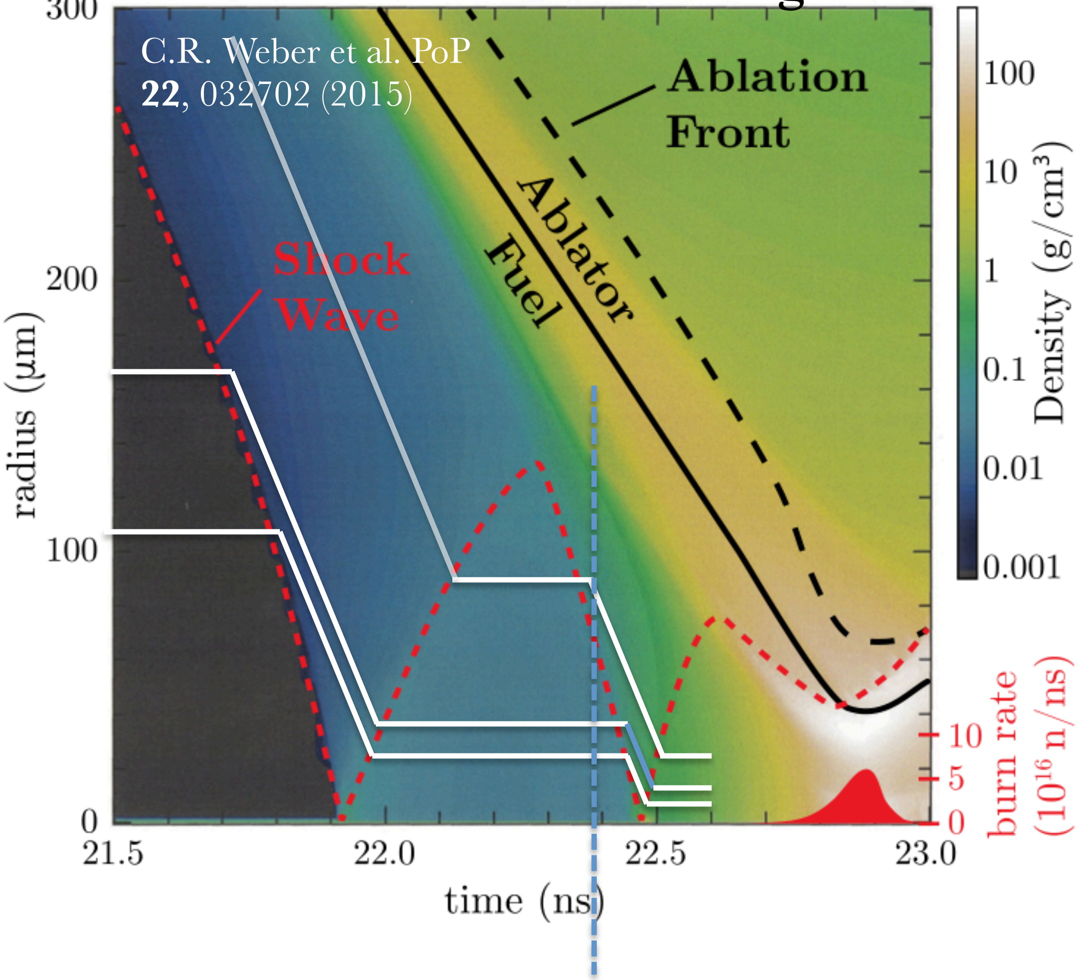
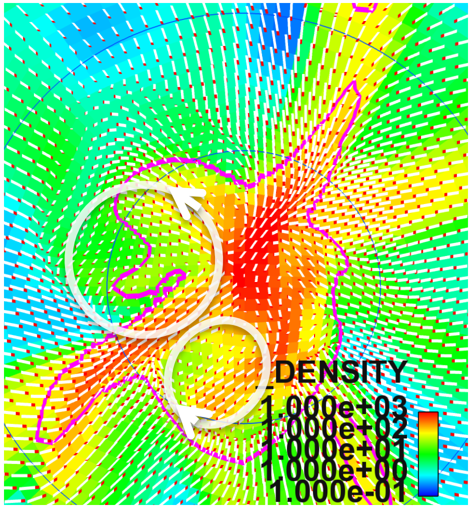


C.R. Weber et al. PoP **22**, 032702 (2015)

t_4(transmitted shock)

Could vortices be compressed and amplified by transmitted shock?



However, viscosity is more important here than in a Z pinch:

$$\nu_i(\mathrm{cm}^2/\mathrm{s}) = 3.3 \times 10^{-5}\,\frac{\sqrt{A}\,[T(\mathrm{eV})]^{5/2}}{\ln\Lambda\,\boxed{Z^4}\,\rho(\mathrm{g}/\mathrm{cm}^3)}$$

How important is centrifugal force in ICF capsules?

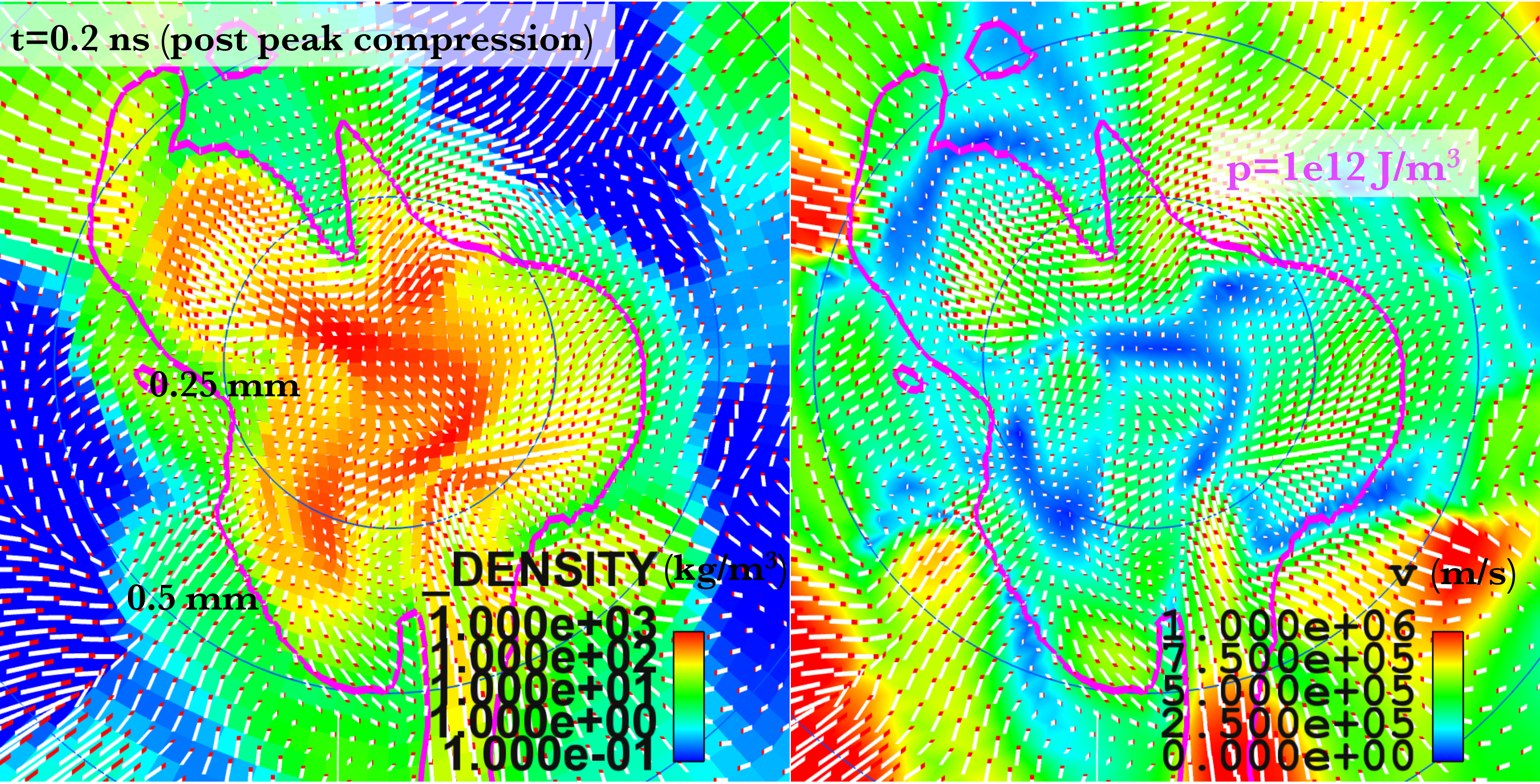
Equation of motion:

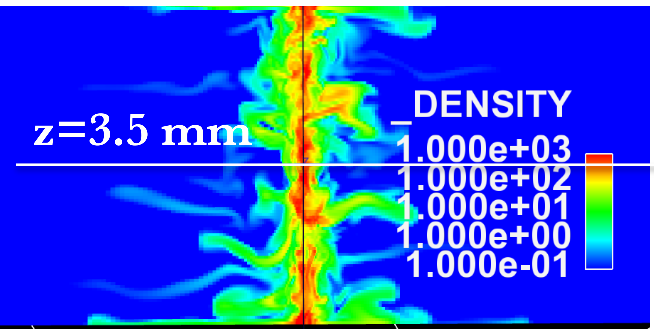
$$\rho\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right)=-\nabla p\ .$$

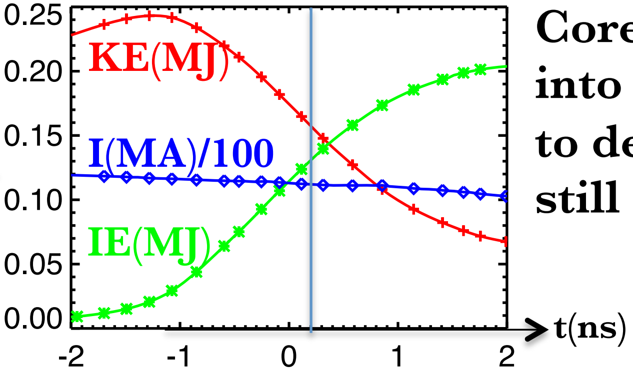
radial component (spherical geometry):

$$\rho\frac{\partial v_r}{\partial t}=-\underbrace{\frac{\partial p}{\partial r}}_{\text{pressure gradient}}-\rho\left[v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi}-\underbrace{\frac{v_\theta^2+v_\phi^2}{r}}_{\text{centrifugal force}}\right]$$

3D simulation fluid flows







Core plasma expands outward into the imploding plasma due to decreasing p_{ram}. Total KE is still dropping.

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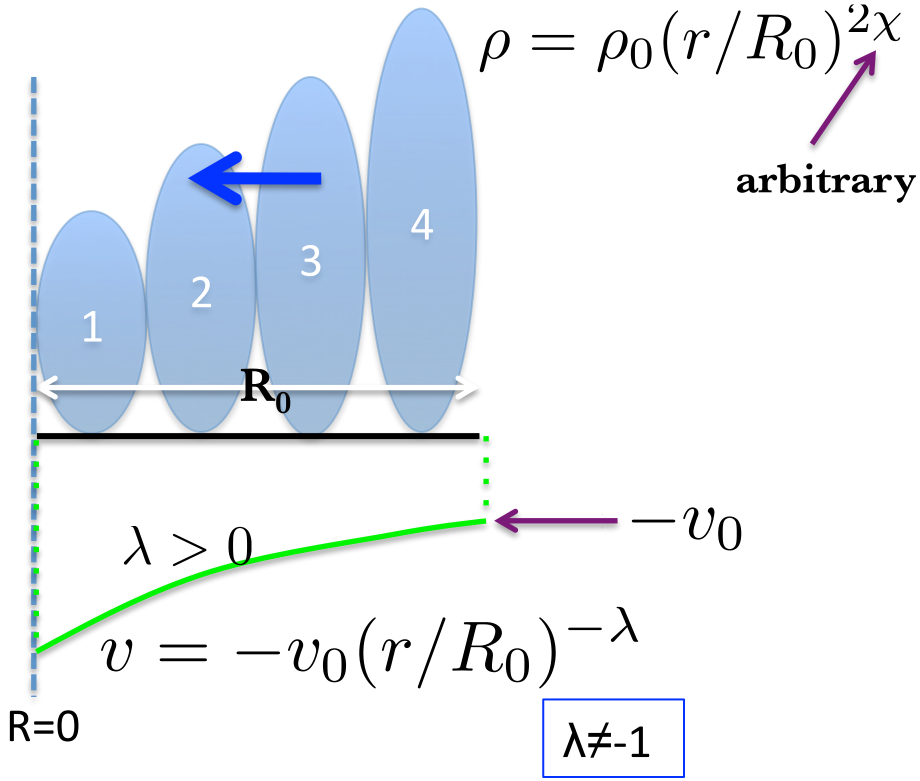
Can a 1D shock solution describe 3D stagnation?



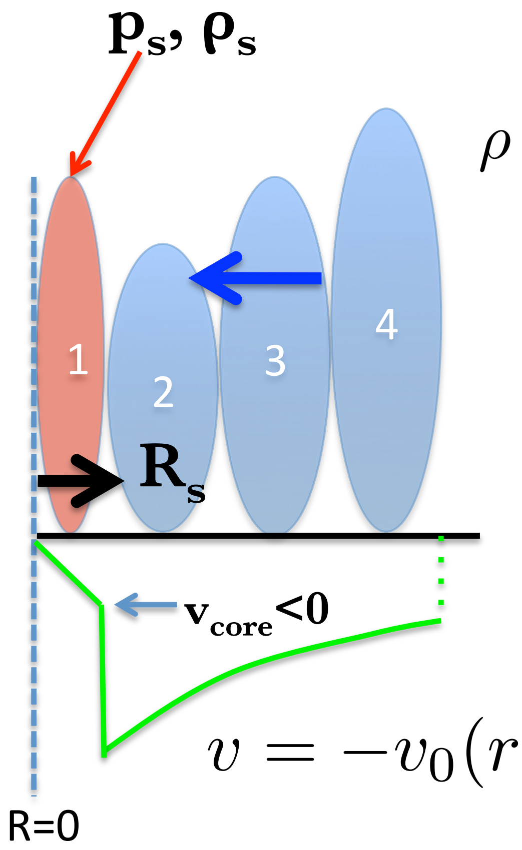
1D shock model has been useful in providing qualitative explanation.

How about *quantitative* comparison?

Use the generalized shock solution (A. Velikovich), allowing non-uniform initial ρ(r), v(r).

E.P. Yu, A.L. Velikovich, and Y. Maron, PoP **21**, 082703 (2014)

Can a 1D shock solution describe 3D stagnation?



$$\rho = \rho_0 (r/R_0)^{2\chi}$$

$$v = -v_0 (r/R_0)^{-\lambda}$$

1D shock model has been useful in providing qualitative explanation.

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E.P. Yu, A.L. Velikovich, and Y. Maron, PoP **21**, 082703 (2014)

$$p_s(t) \propto t^{\frac{2(\chi-\lambda)}{1+\lambda}}$$

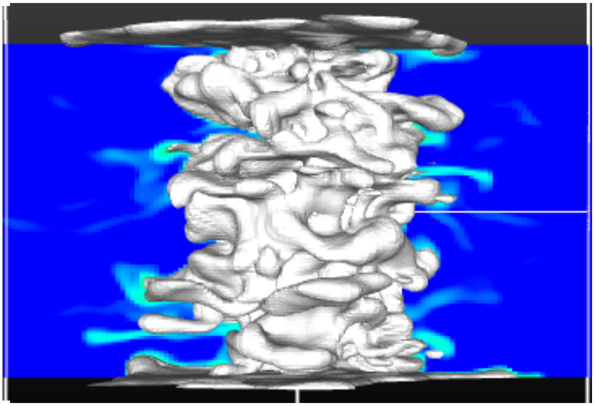
$$\rho_s(t) \propto t^{\frac{2\chi}{1+\lambda}}$$

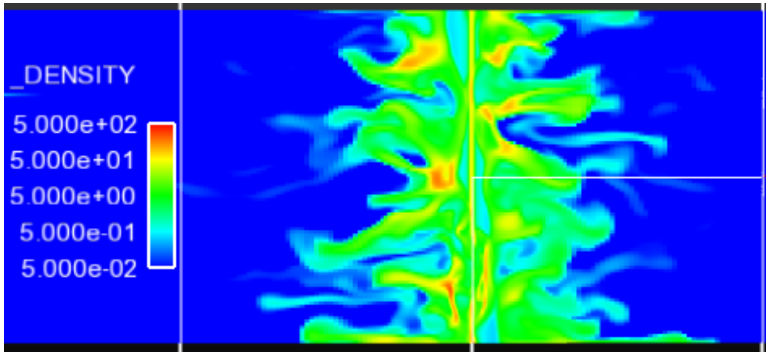
$$R_s(t) \propto t^{\frac{1}{\lambda+1}}$$

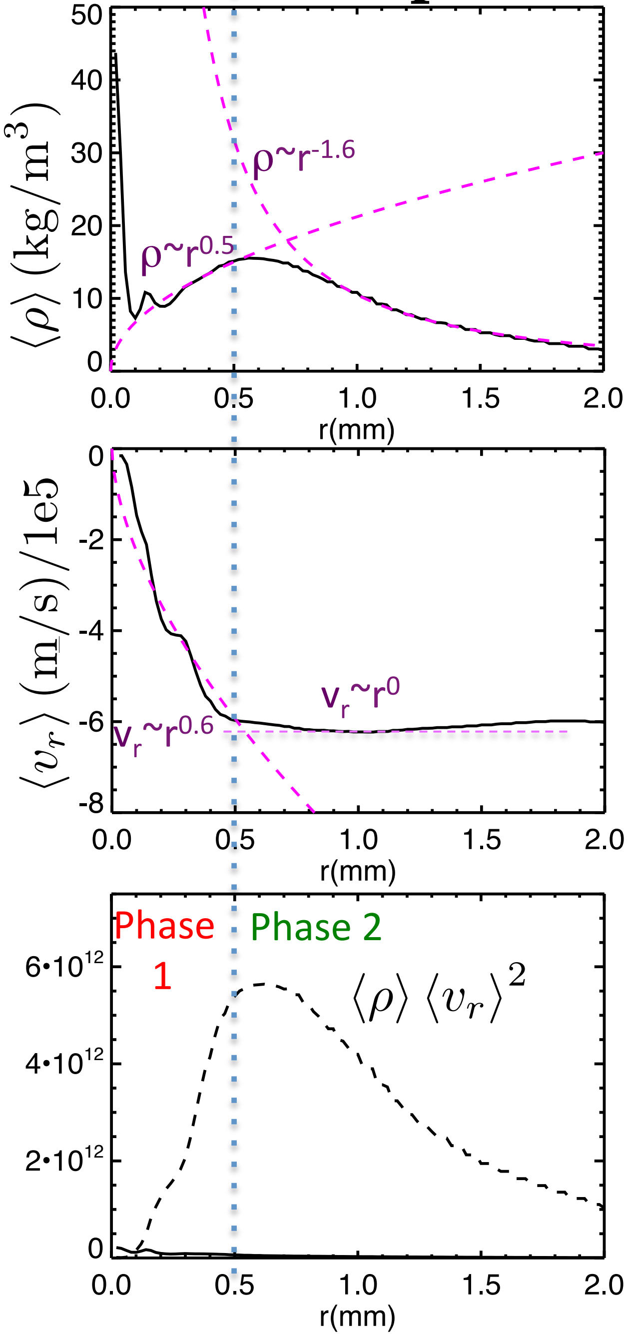
Pre-stagnation profiles exhibit "2 phase" profile

t=-1.6 ns

ρ=1 kg/m³ contour







Phase 1:

ρ_s~$t^{1.25}$

p_s~$t^{4.25}$

R_s~$t^{5/2}$

Phase 2:

ρ_s~$t^{-1.6}$

p_s~$t^{-1.6}$

R_s~t

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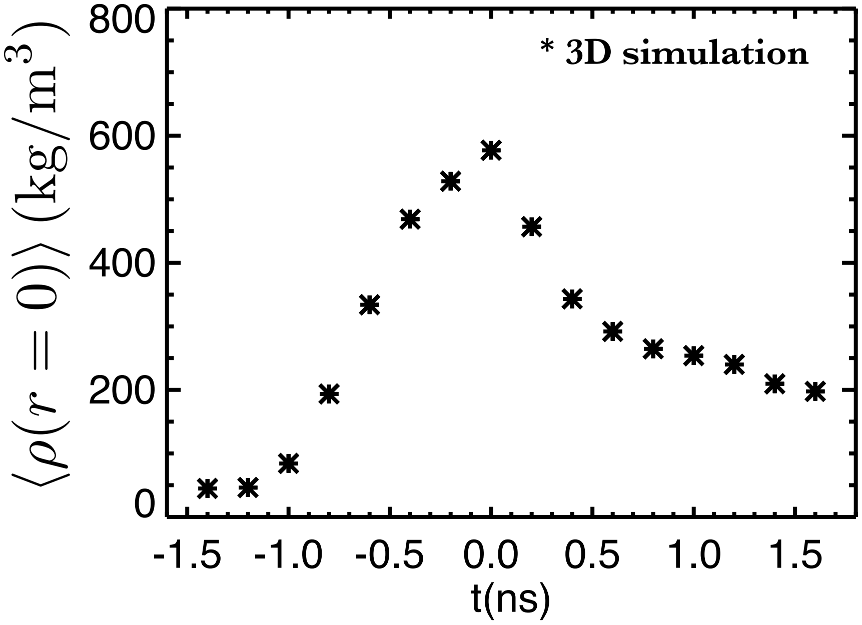
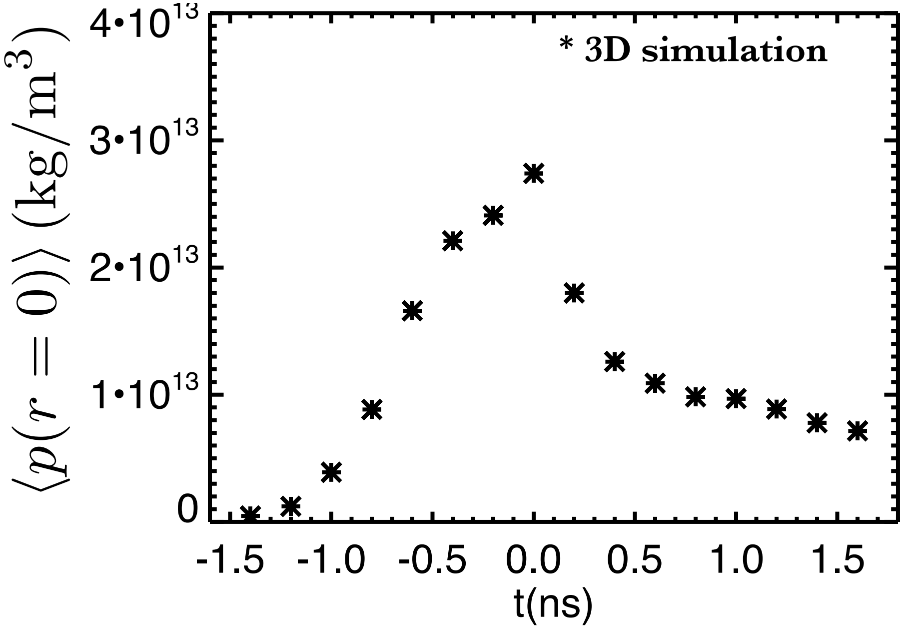
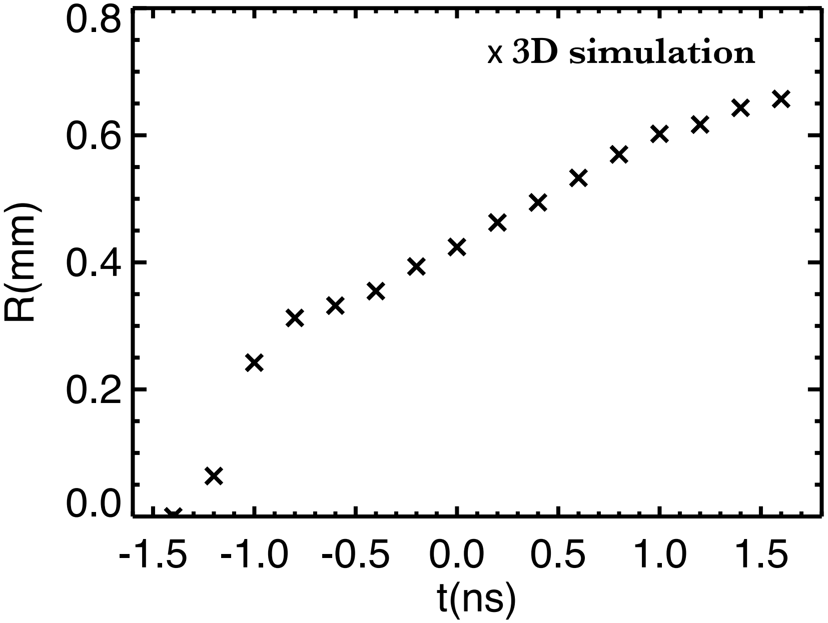
Comparison of shock solution with 3D simulation







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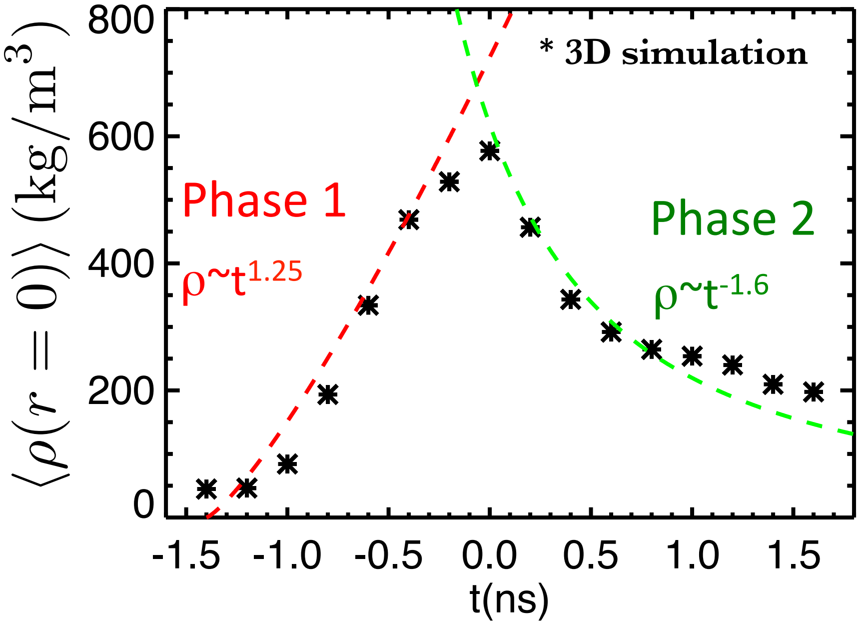
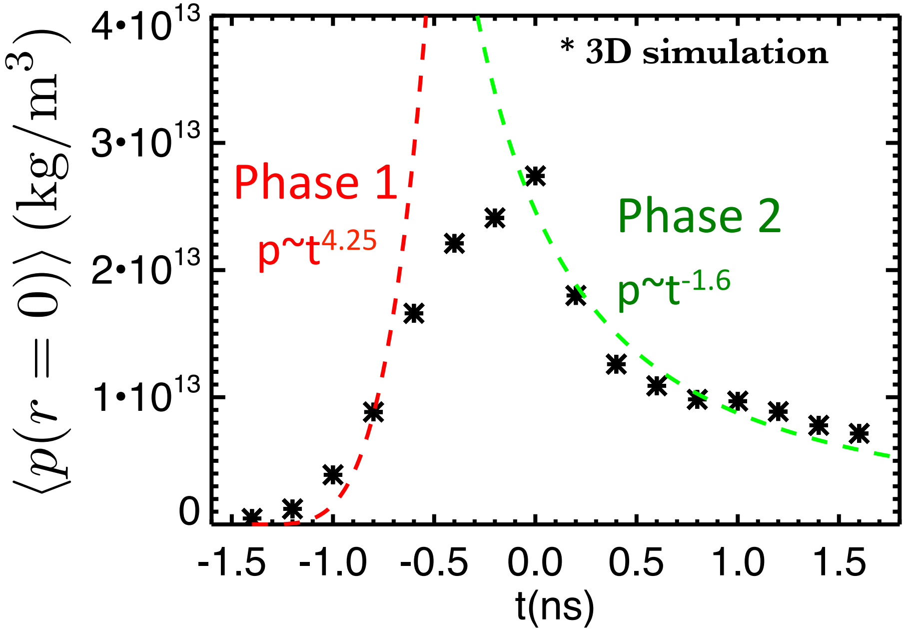
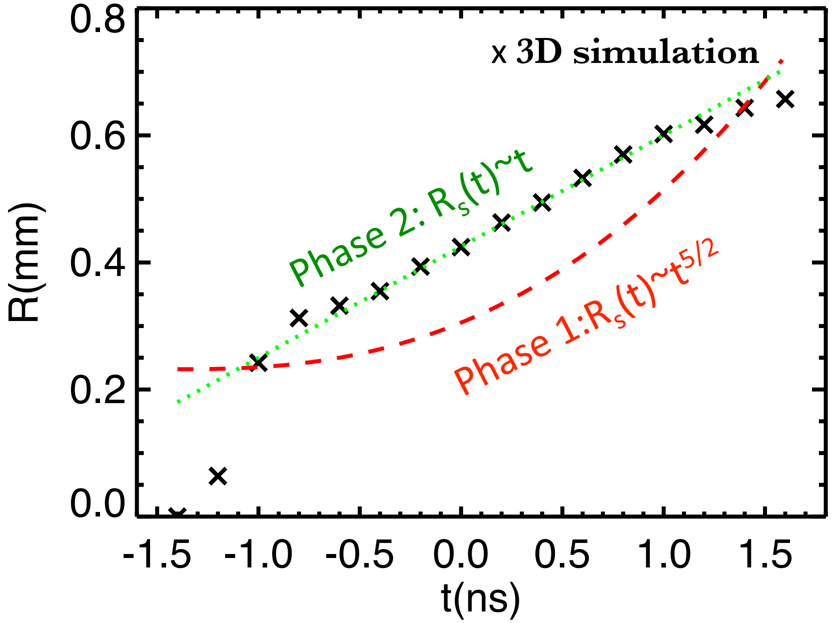
Shock solution agrees with 3D simulation during “phase 2”







Phase 1 describes initial accumulation of mass on axis. There isn’t sufficient symmetry for a 1D shock picture to be valid.

During **phase 2**, there is already a “large” core on axis, increasing the validity of a 1D shock model.

How might the residual kinetic energy during phase 1 dissipate?

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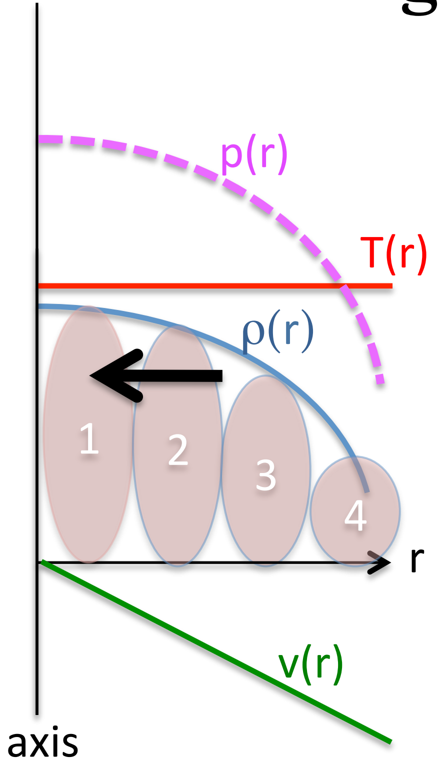
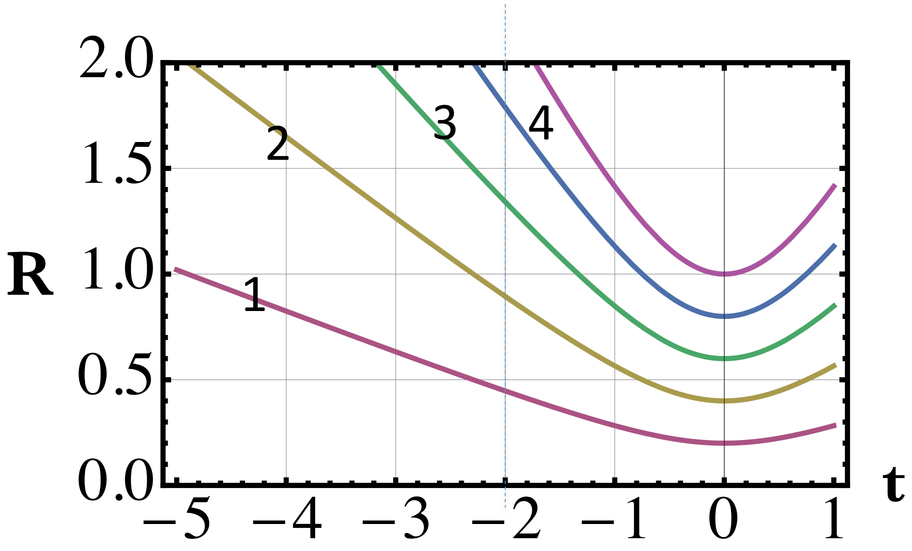
Homogeneous (shockless) stagnation

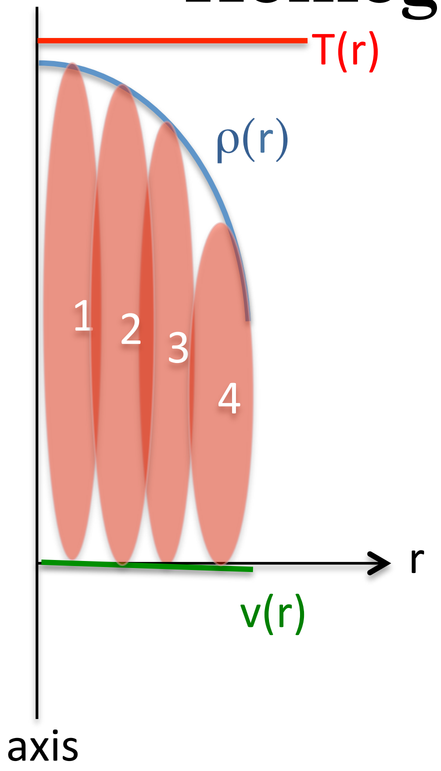


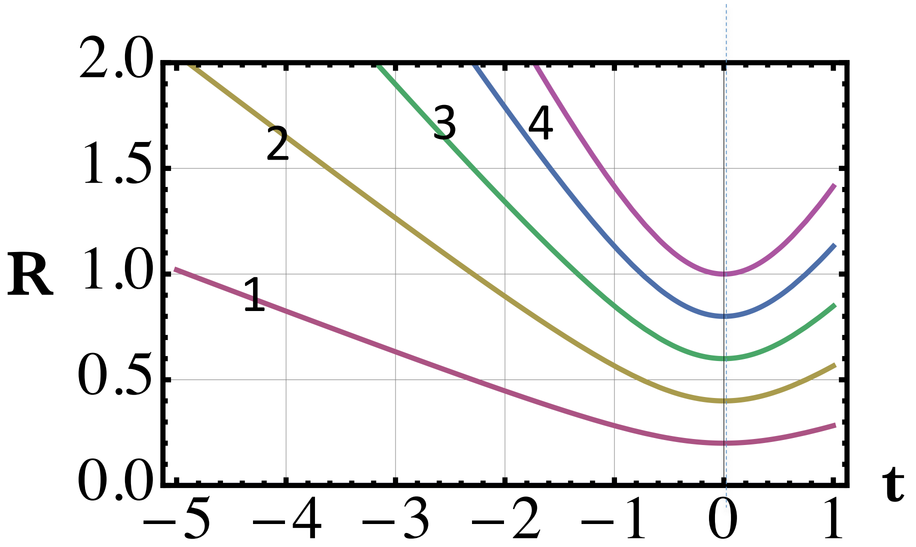


v(r) is linear. This allows particles to compress in unison, without shocks.

Finite T is allowed. This results in a finite pressure gradient, causing particles to gradually slow down.

Homogeneous (shockless) stagnation





Stagnation is complete. All kinetic energy has converted to internal energy. The key parameter:

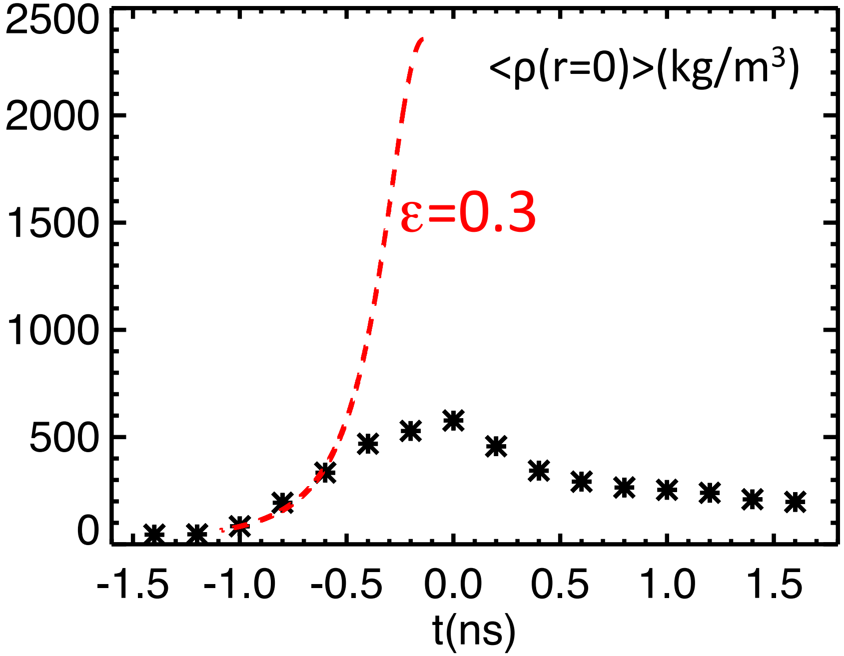
$$\epsilon = \frac{\int p dV}{\int \frac{1}{2}\rho v^2 dV}$$

Initial thermal energy

Initial kinetic energy

Higher ε (thermal energy)➔ less compression

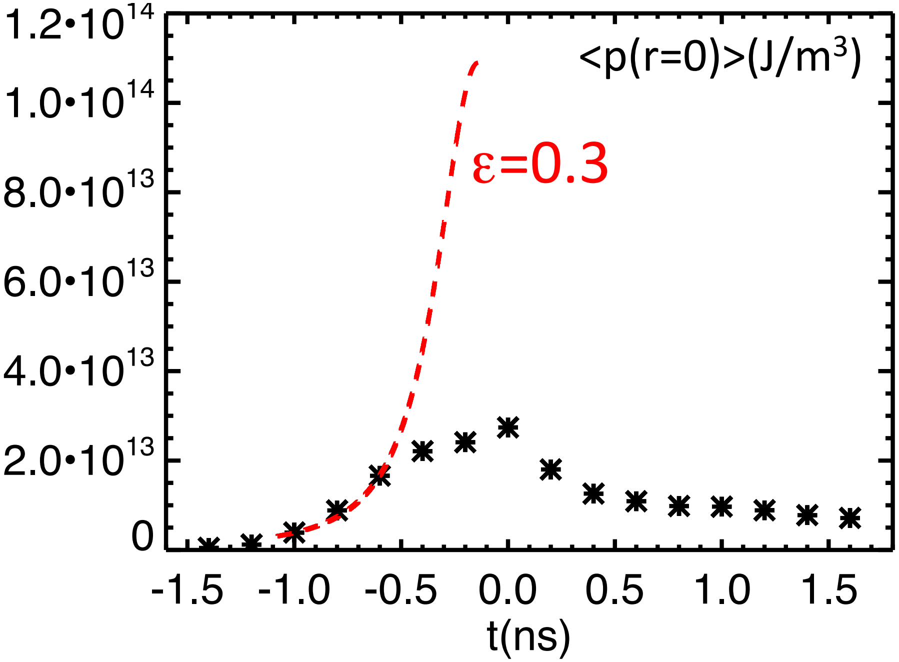
Comparison of homogeneous stagnation with 3D simulation



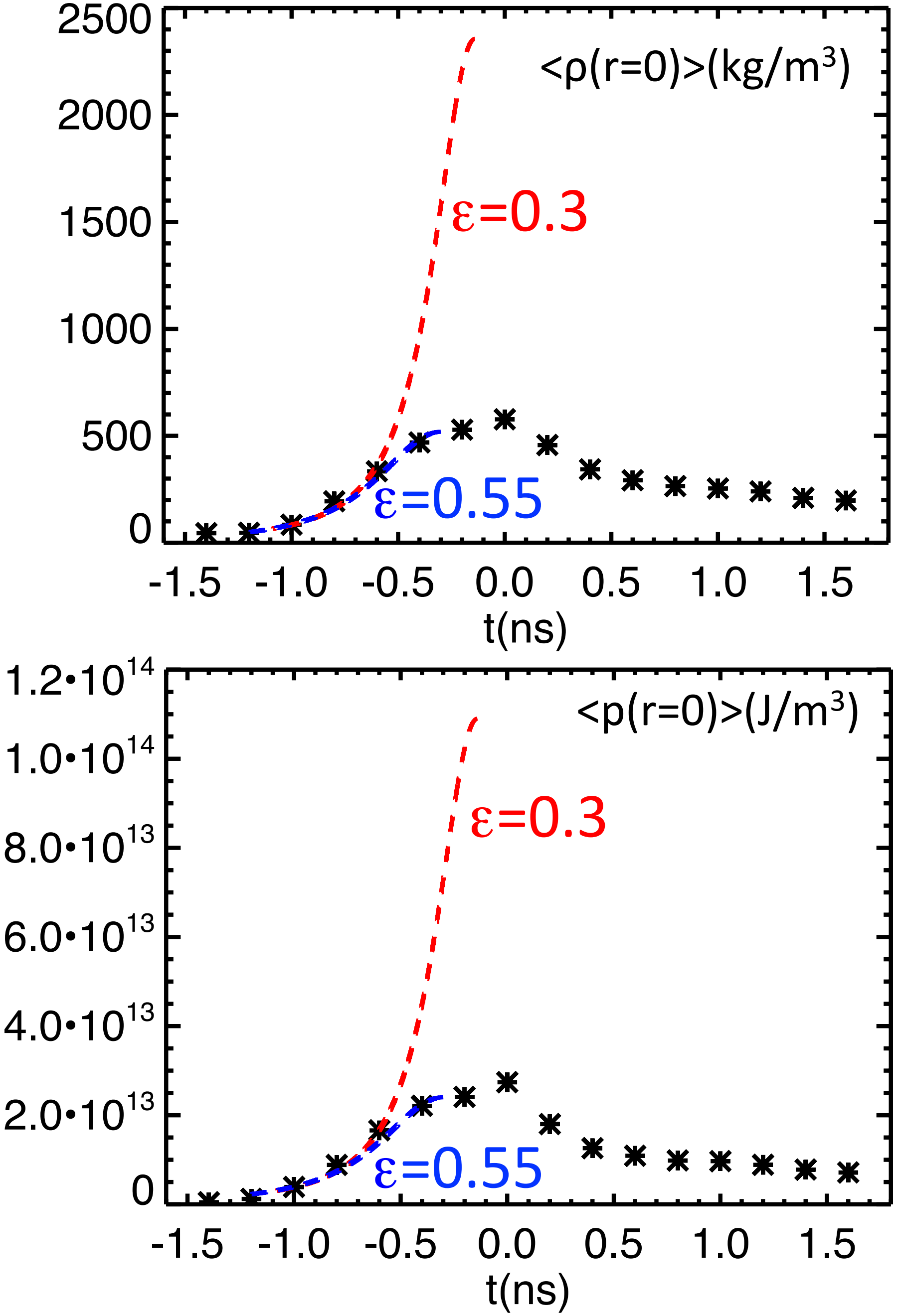
$$\epsilon = \frac{\int p dV}{\int \frac{1}{2}\rho v^2 dV}$$

Computing ε from simulation, theory predicts too high a compression.

But suppose we artificially enhance ε, to account for the centrifugal pressure from the vortices.



Comparison of homogeneous stagnation with 3D simulation



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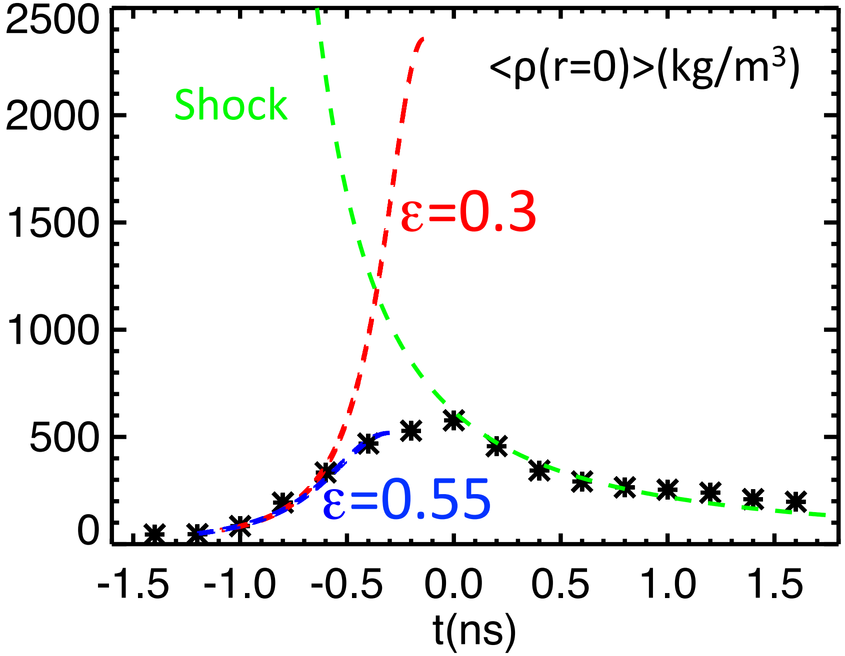
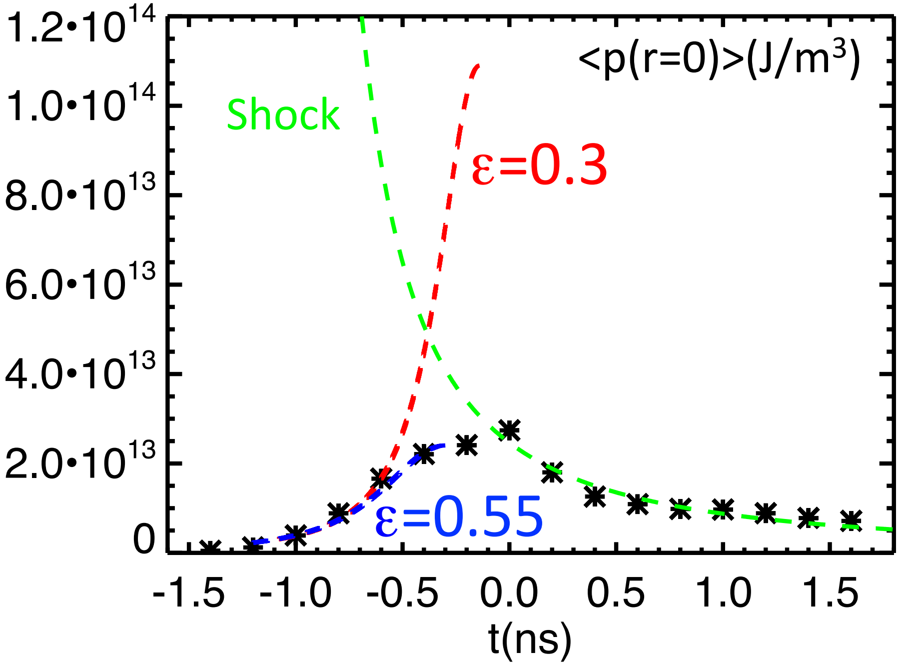
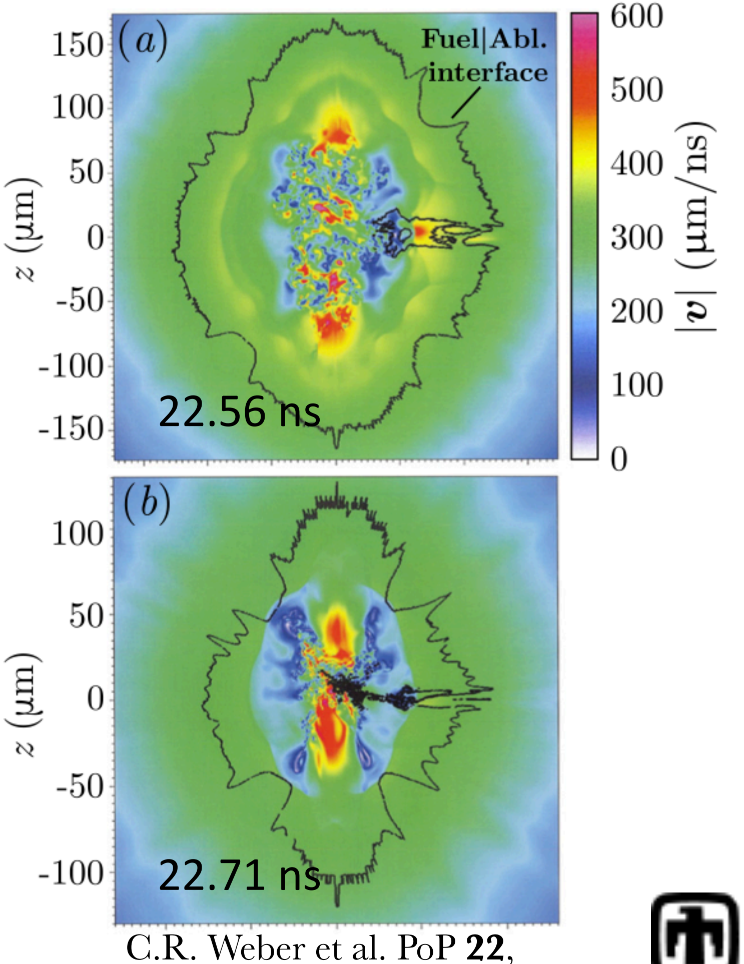
Comparison of homogeneous stagnation with 3D simulation





Strong polar jets probably invalidate use of homogeneous stagnation solution in NIF capsule.

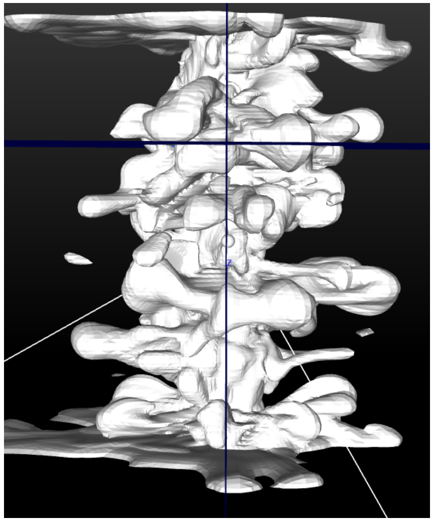


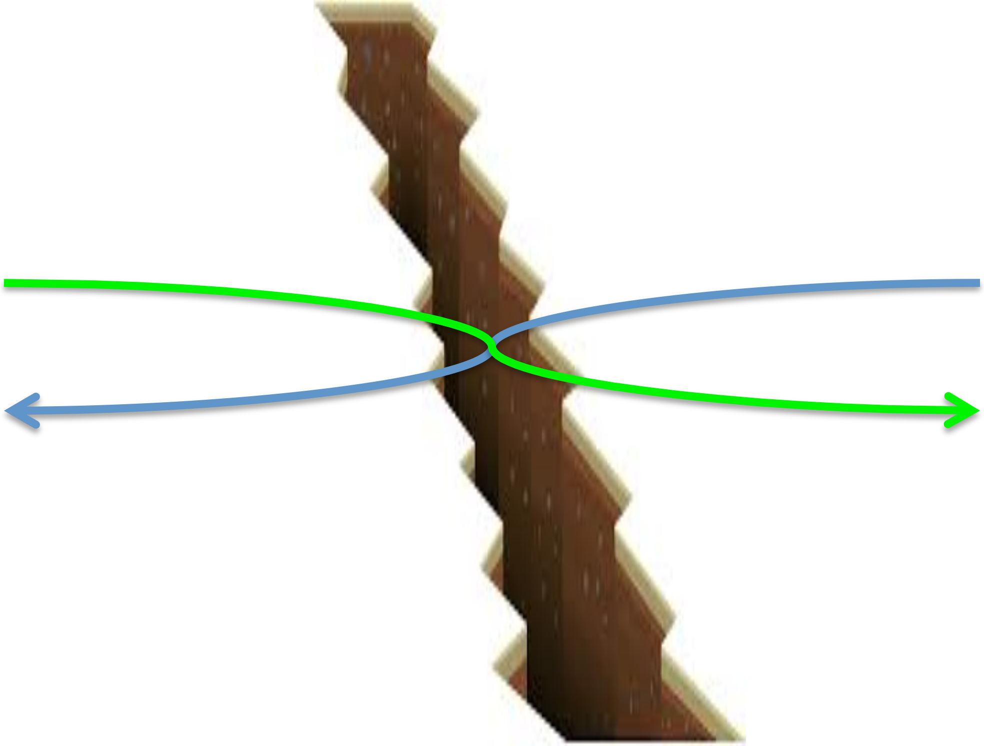
C.R. Weber et al. PoP **22**, 032702 (2015)

Making connections between 3D simulation and 1D theory is beneficial

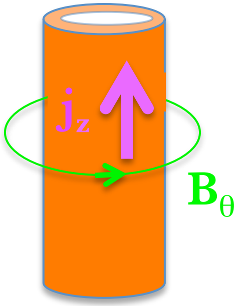
3D

(Z pinch)





1D



Questions:

1. **What 1D physics is important in driving residual flow at stagnation?**

   ram pressure profile

2. **Effects of residual flow on pressure/energy balance at stagnation?**

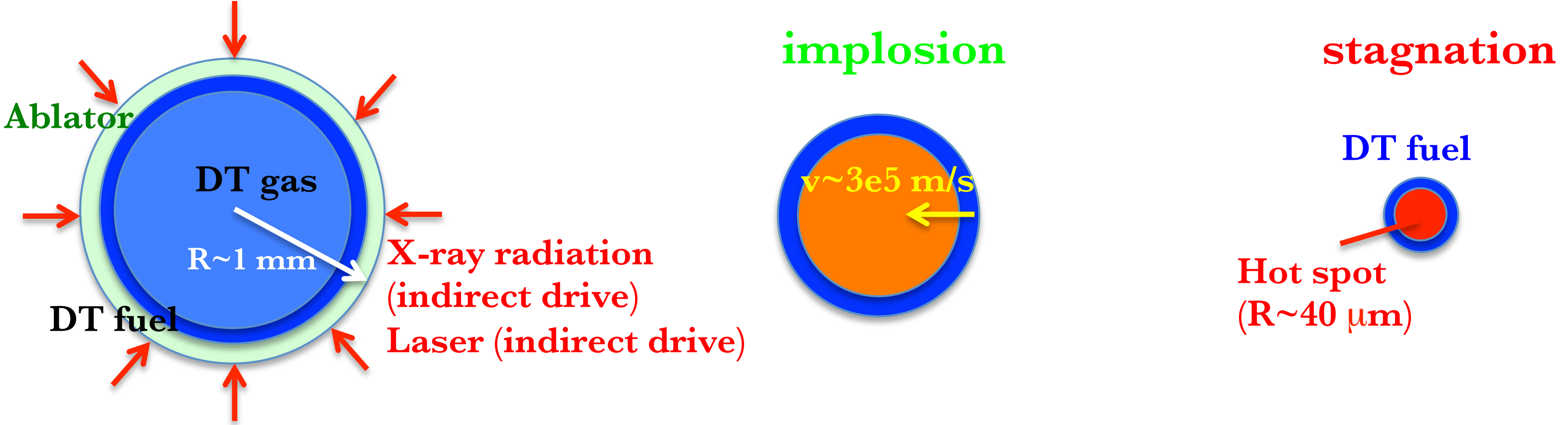
   centrifugal force due to vortices is comparable to pressure gradient

3. **Can a 1D model approximately describe 3D stagnation?**

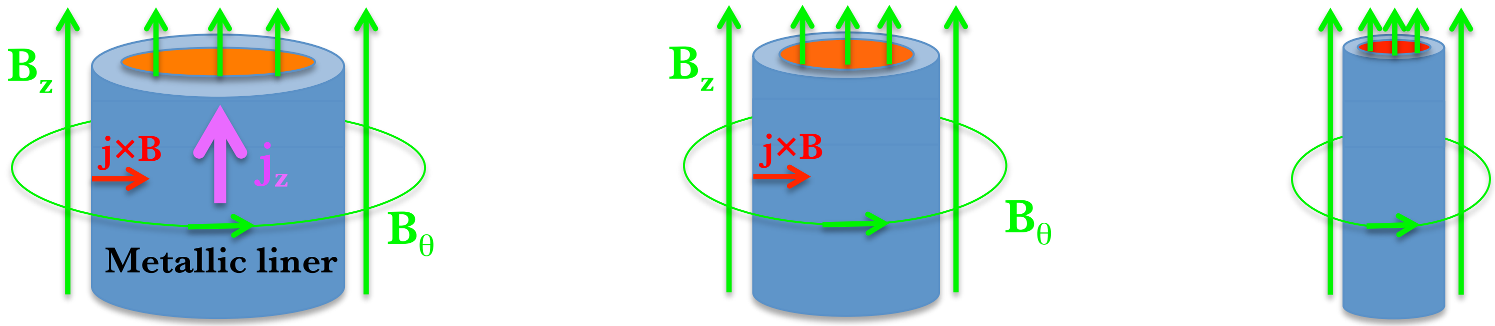
   Shock and shockless stagnation models are helpful in describing $<\rho(t)>$,$<p(t)>$,$R(t)$

Backup: wire array ablation

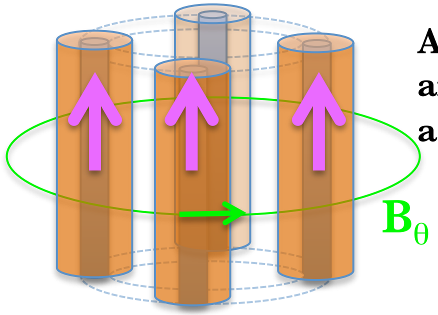
Spherical: inertial confinement fusion (ICF) capsule



Cylindrical: MagLIF (ICF)



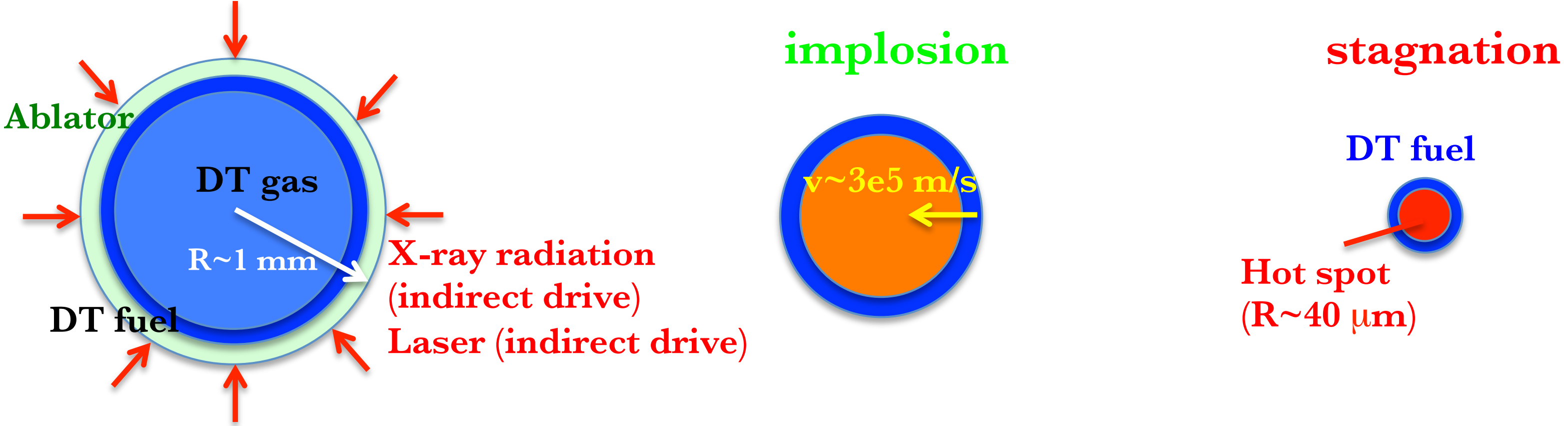
Cylindrical: wire array Z pinch (non-ICF)



A hot, low density plasma corona forms around the wires, supporting current as well as j×B force

Backup: wire array ablation

Spherical: inertial confinement fusion (ICF) capsule



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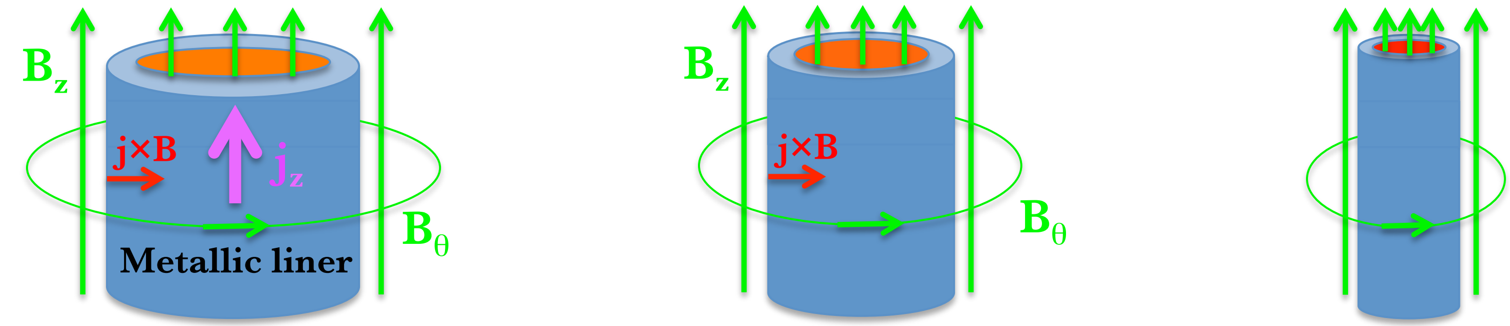
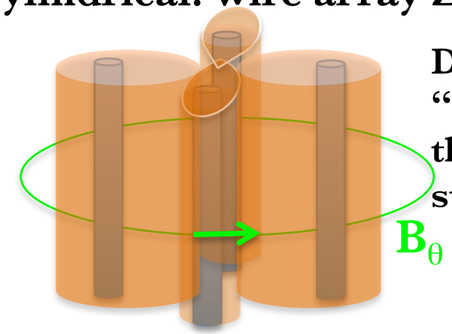


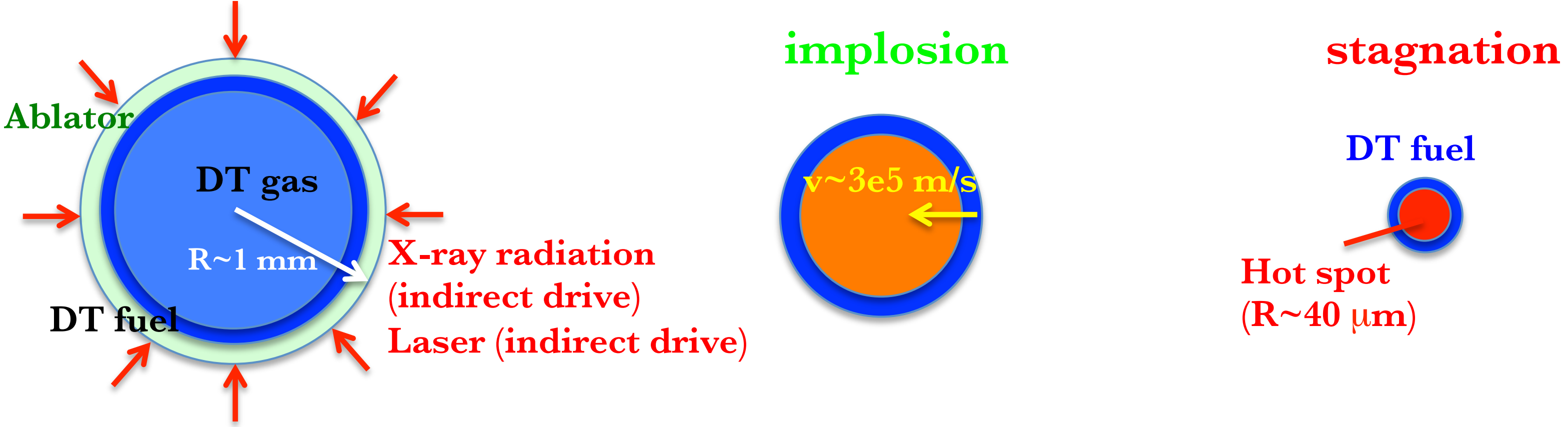
Cylindrical: wire array Z pinch (non-ICF)



During the *ablation phase*, outer edge of wires “cooks off” and converts to corona, which is then flung towards axis. Wires remain stationary

Backup: wire array ablation

Spherical: inertial confinement fusion (ICF) capsule



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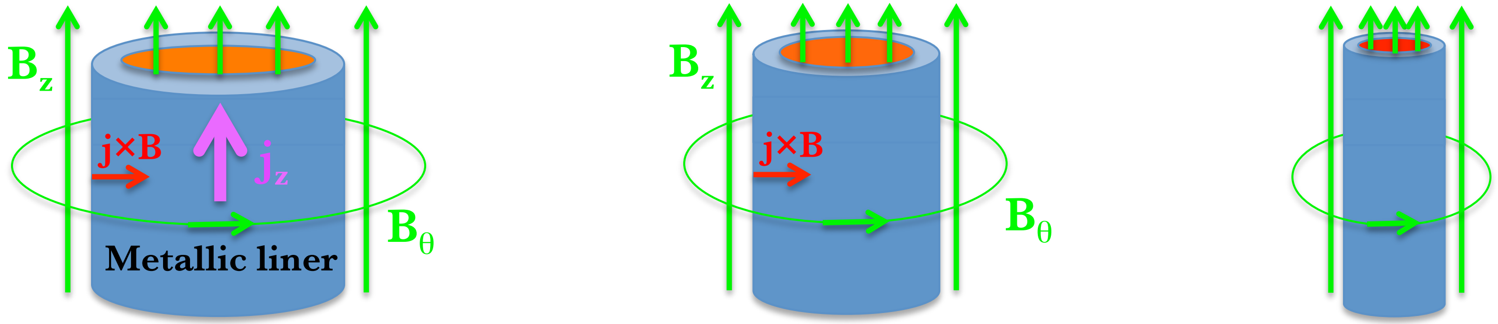
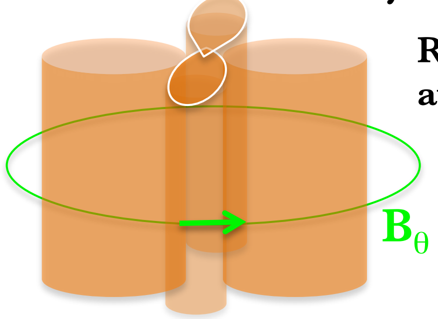


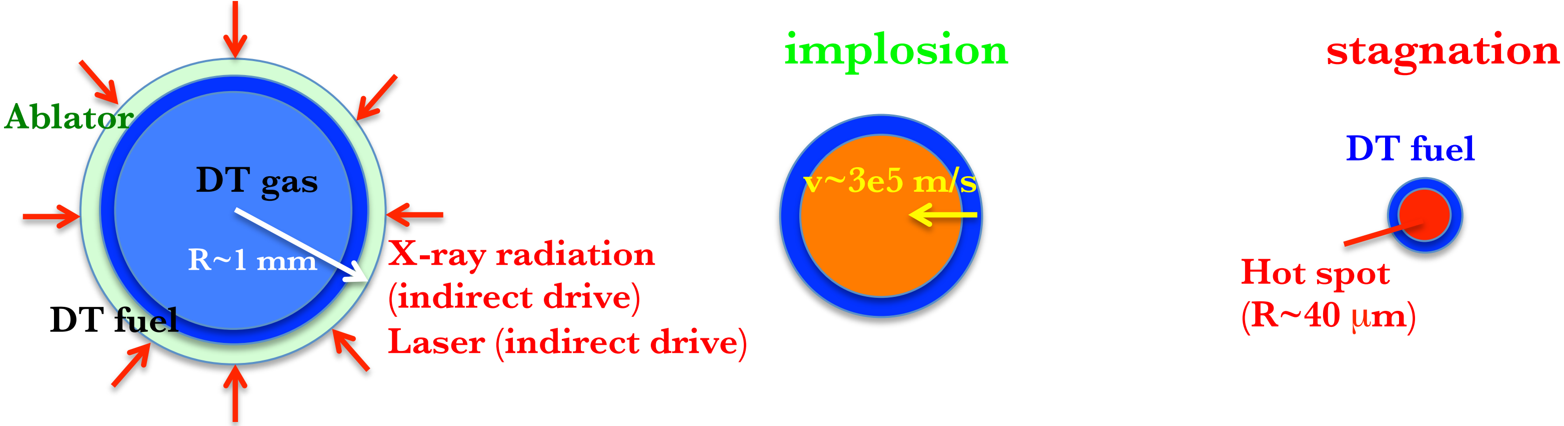
Cylindrical: wire array Z pinch (non-ICF)



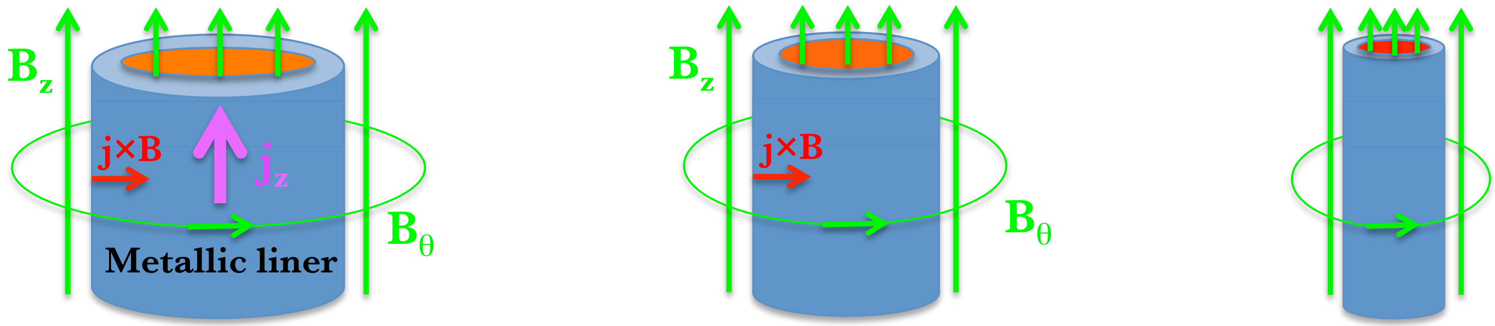
Roughly speaking, once wires have cooked away, the implosion begins

Backup: wire array ablation

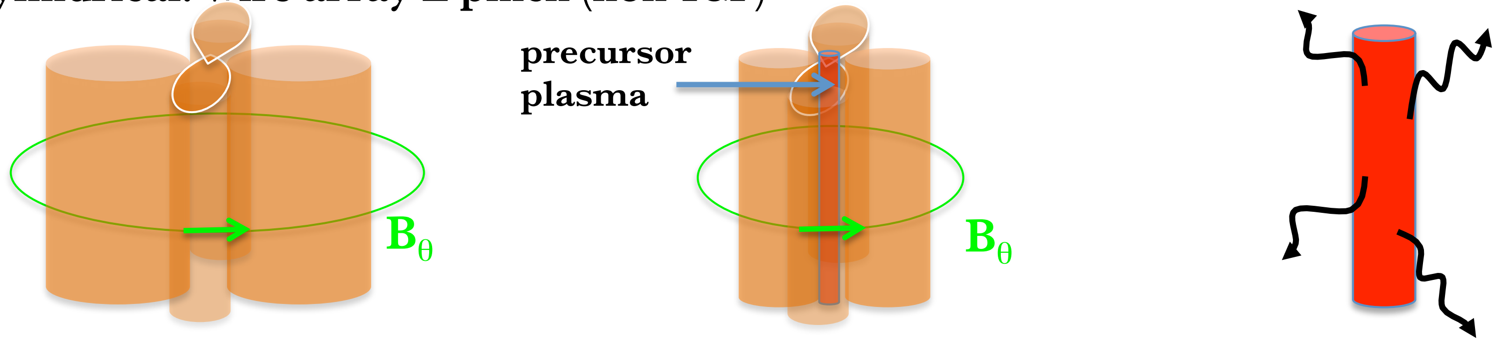
Spherical: inertial confinement fusion (ICF) capsule



Cylindrical: MagLIF (ICF)



Cylindrical: wire array Z pinch (non-ICF)



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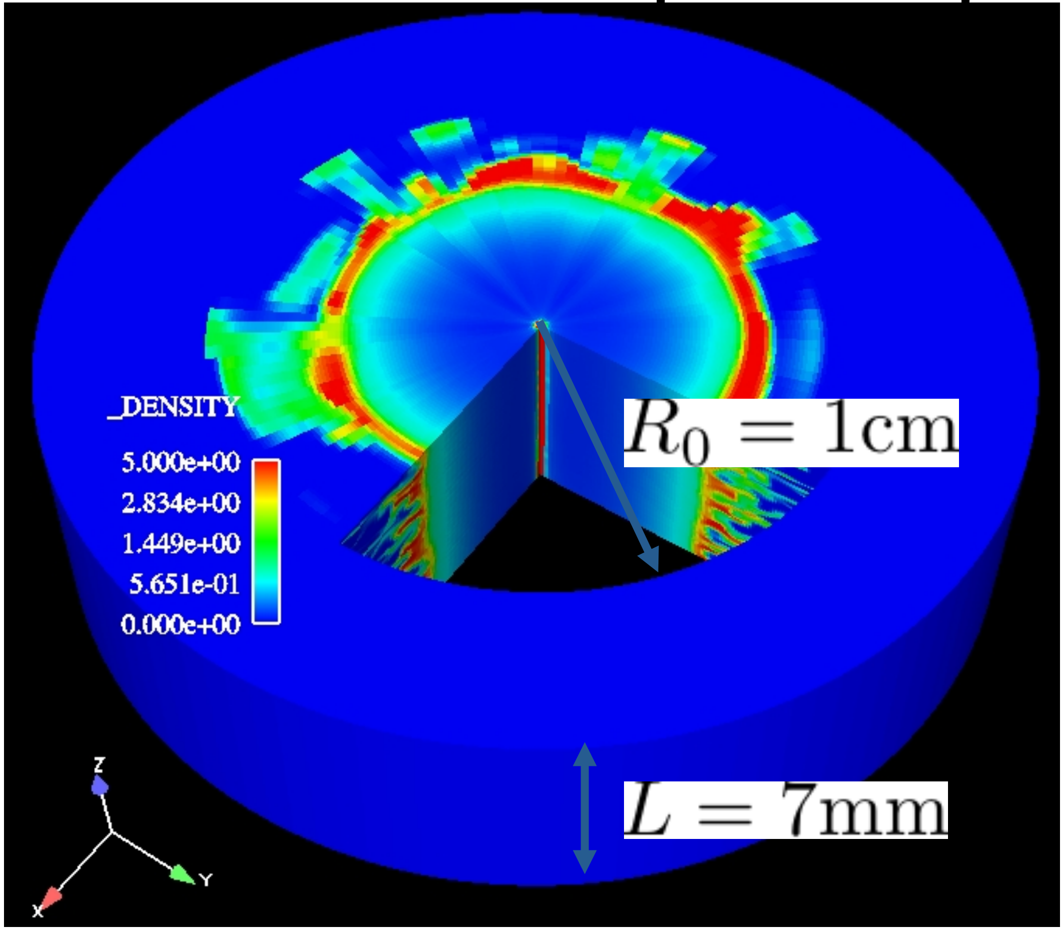
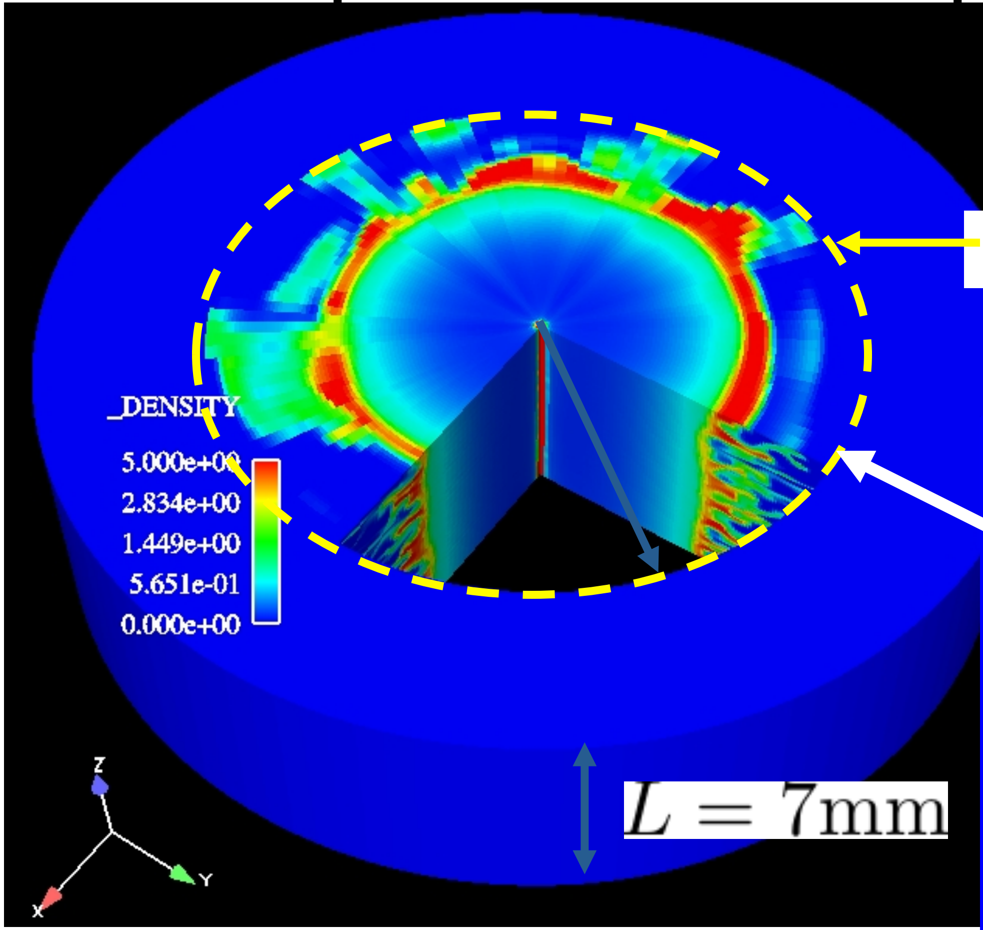
Backup: mass injection scheme



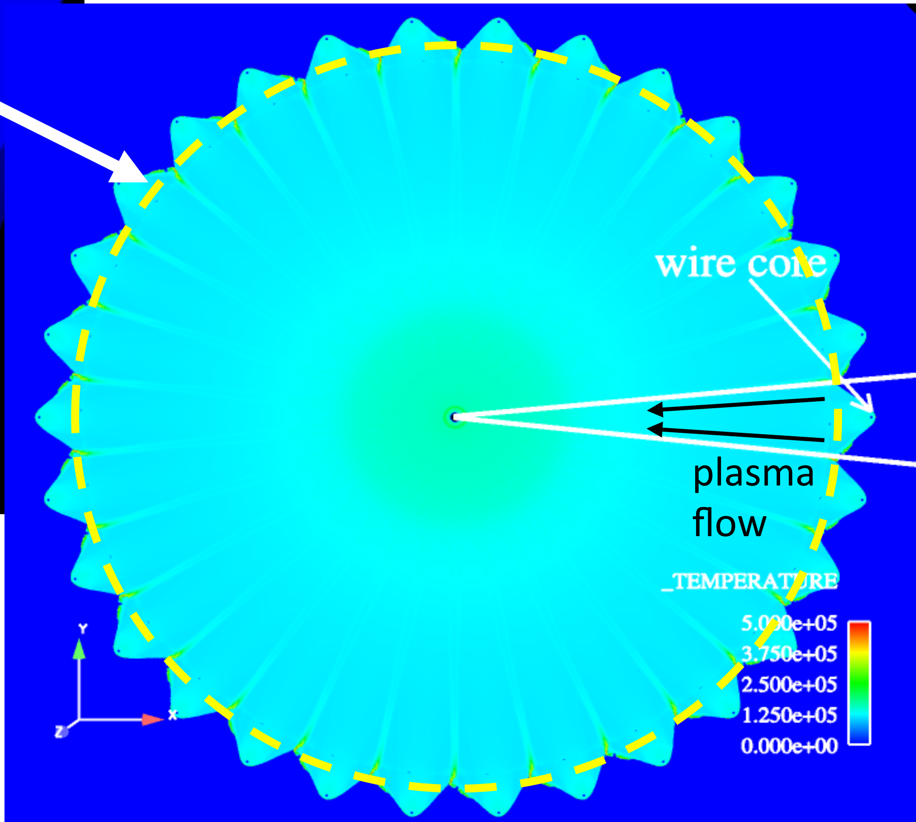
$R_0 = 1\text{cm}$

$L = 7\text{mm}$

Backup: Model ablation/implosion via mass injection

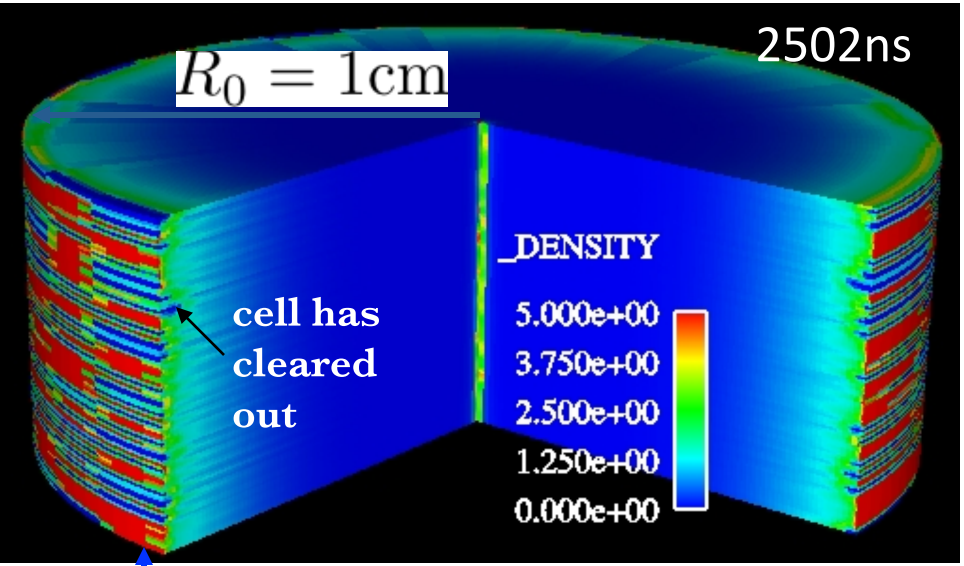


Mass injection surface



This idea has been used before:
J.P. Chittenden, et al., Phys. Plasmas 11, 1118 (2004)
P.V. Sasorov, in V.V. Aleksandrov et al., Plasma Phys. Reports, 27, 89 (2001)

Backup: mass injection parameters constrained by experiment



mass is injected slowly (v~1e4 m/s) at

mass injection surface

Each cell has mass m and ablates according to

$$\dot{m} = \dot{m}_0 (I/I_0)^{\alpha}$$

determines when cell finishes ablating. Determines when array starts to implode.

currently,* α=1.4,

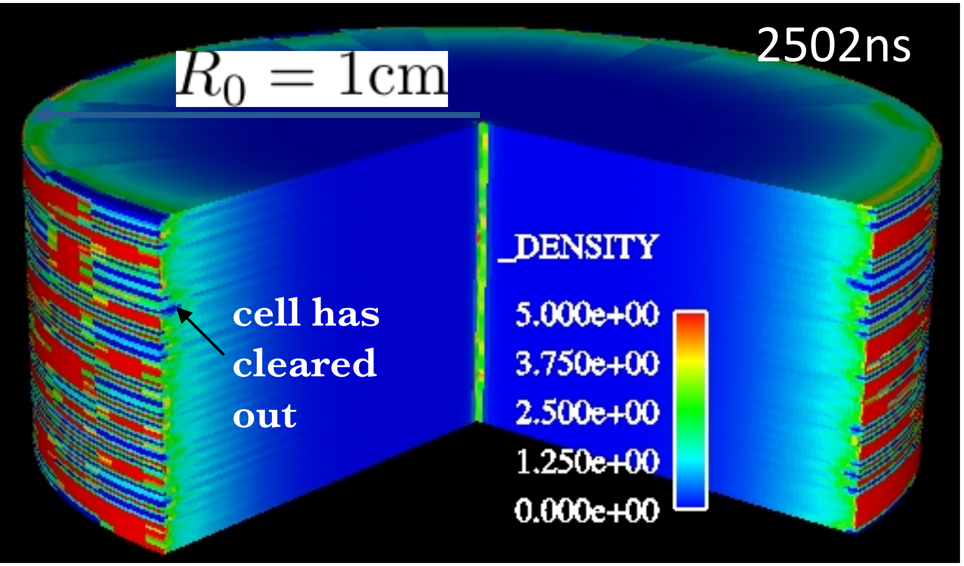
determines distribution of prefill plasma.

---

*E.P. Yu, B.V. Oliver, P.V. Sasorov et. al., Phys. Plasmas 14, 022705 (2007)

Backup: mass injection parameters constrained by experiment

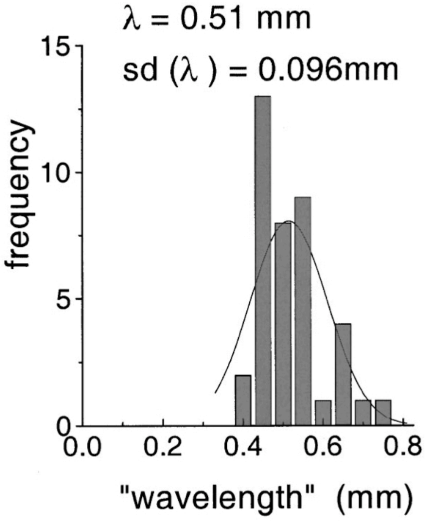
dr~10-20 um, dz~60, Nϕ=120

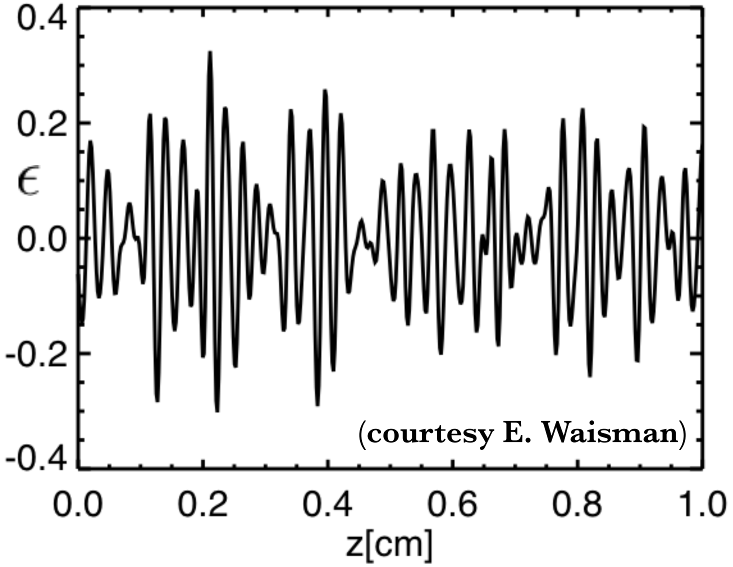


cell has cleared out

$$\dot{m} = \dot{m}_0 (I/I_0)^{\alpha} (1 + \epsilon(z))$$

spectral content determined by experimental histogram. Amplitude constrained by contrast ratio between streams.

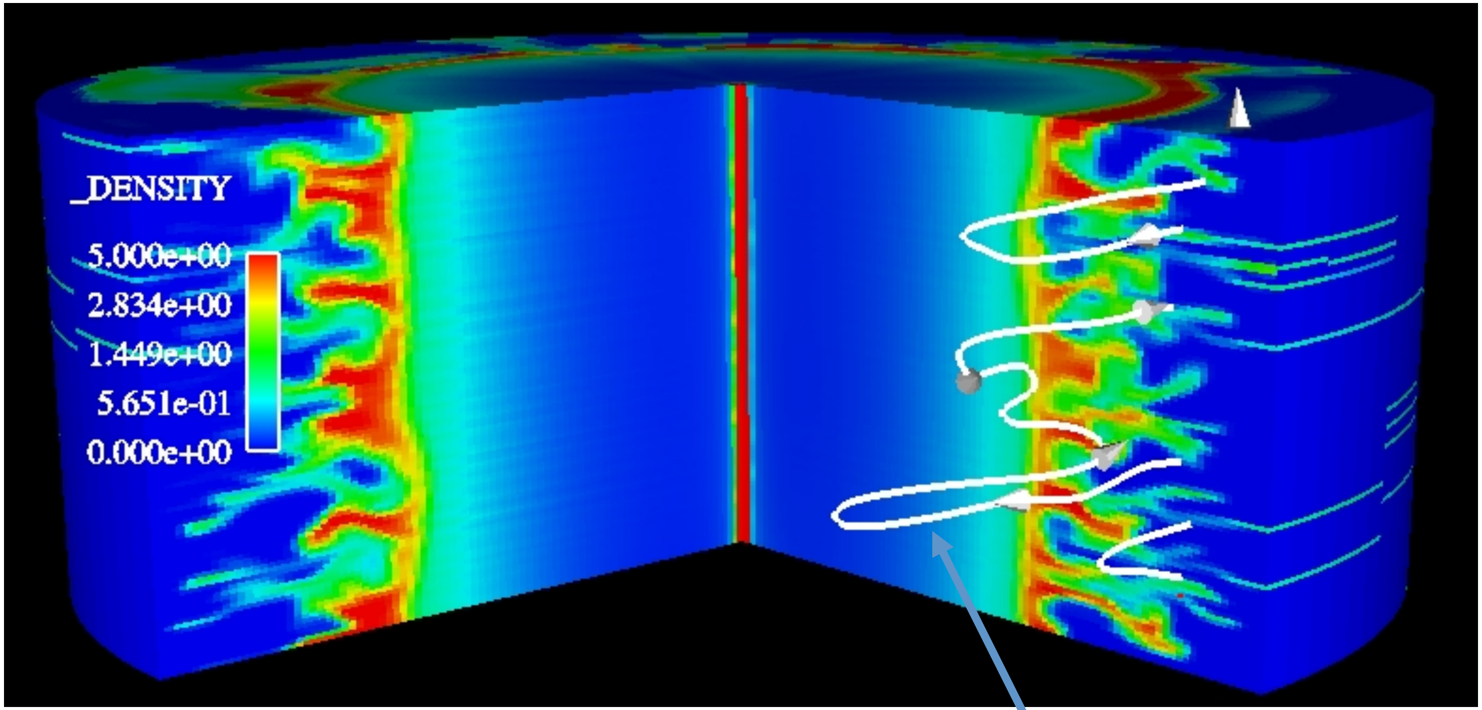




(courtesy E. Waisman)

Backup: 3D structure during implosion

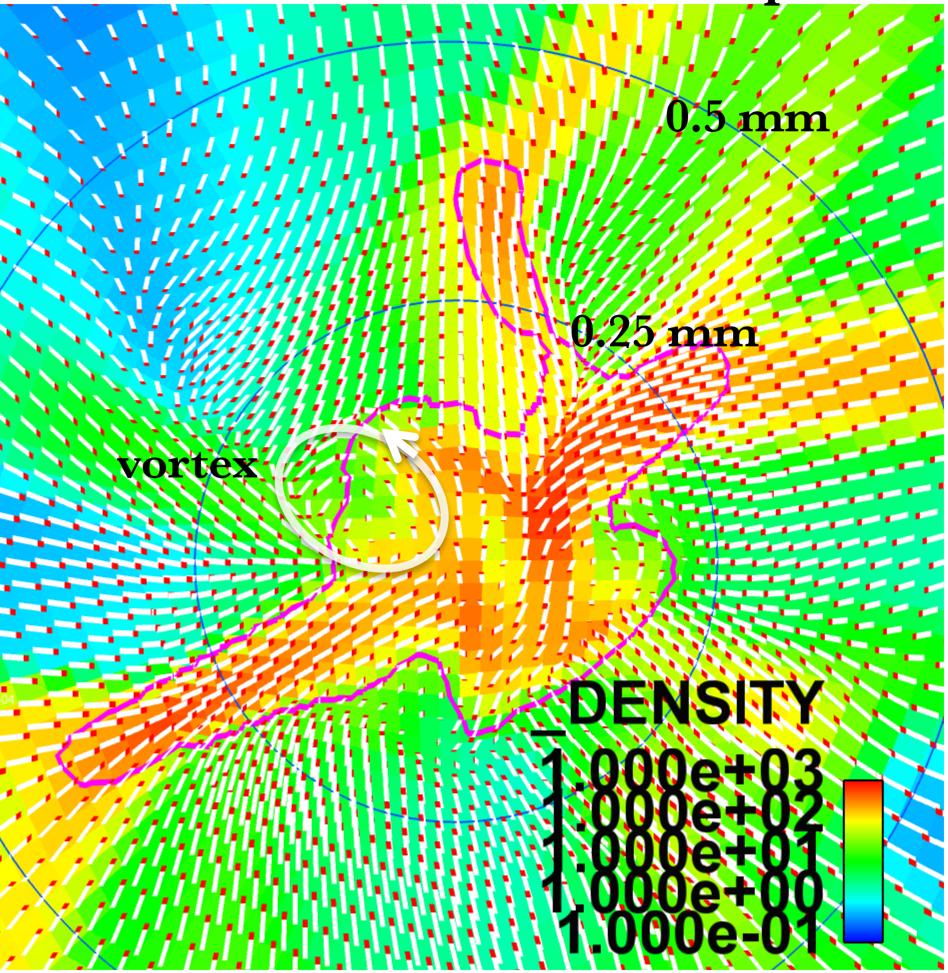
C=3%, t=2518 ns

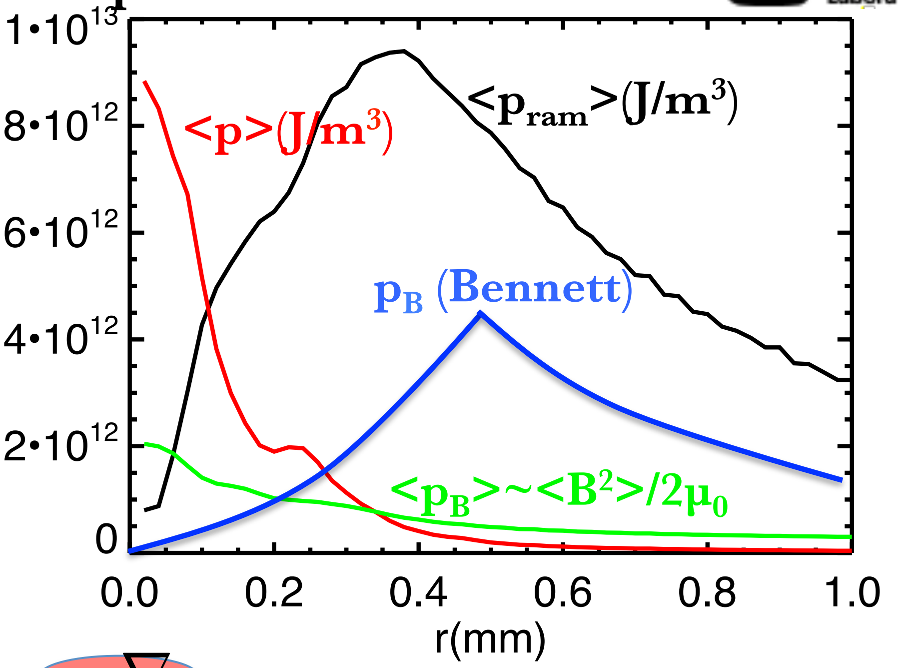


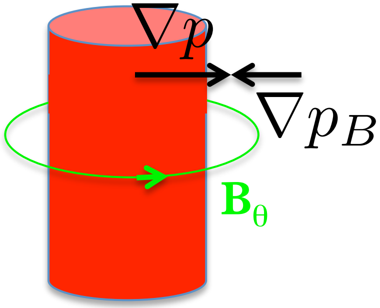
current streamline

t=-0.8 ns

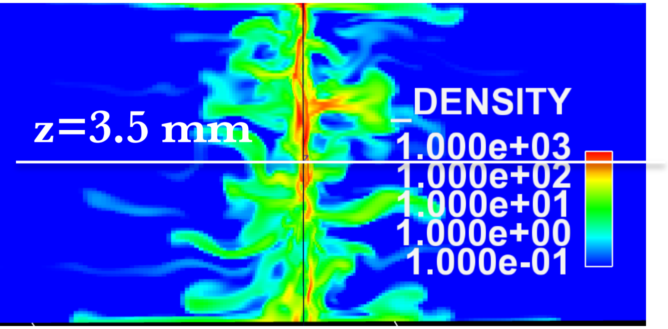
Backup: Bennett equilibrium

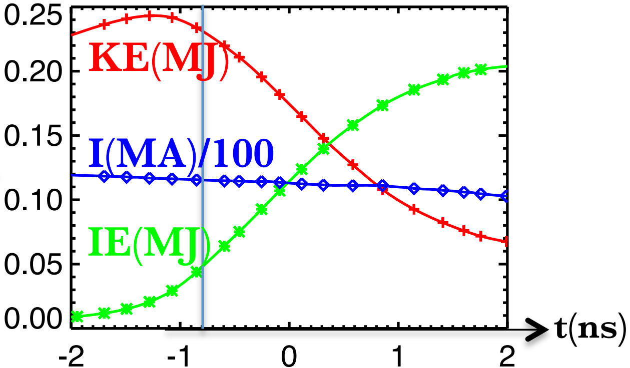






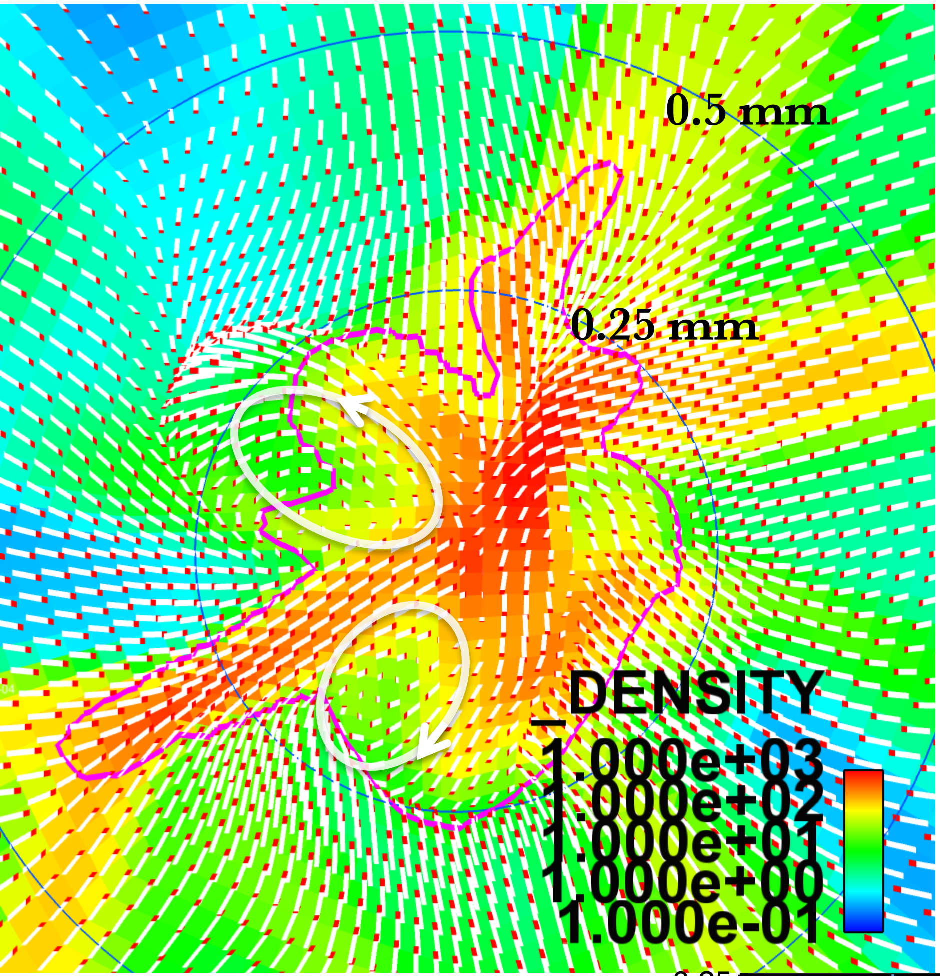
This is very different from a Bennett equilibrium

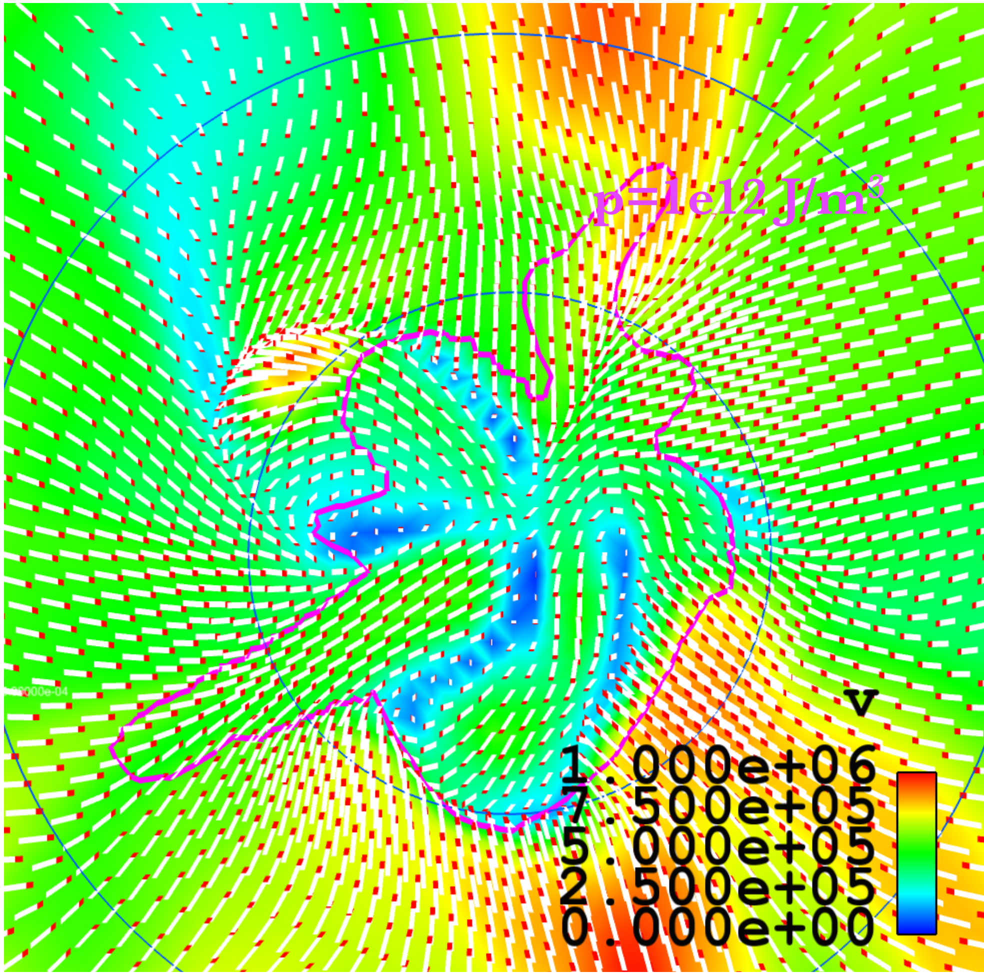


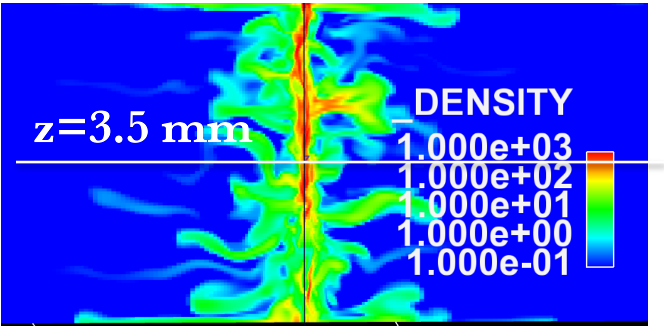


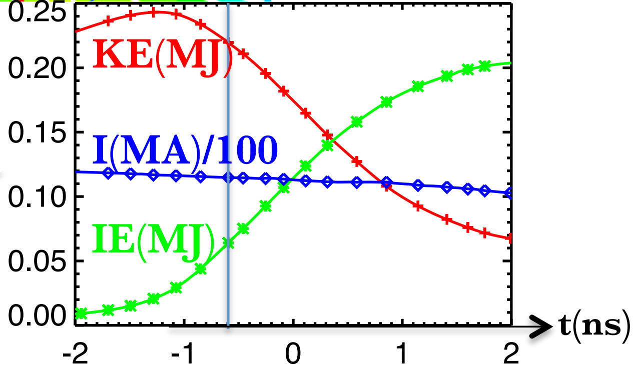
t=-0.6 ns

3D simulation fluid flows



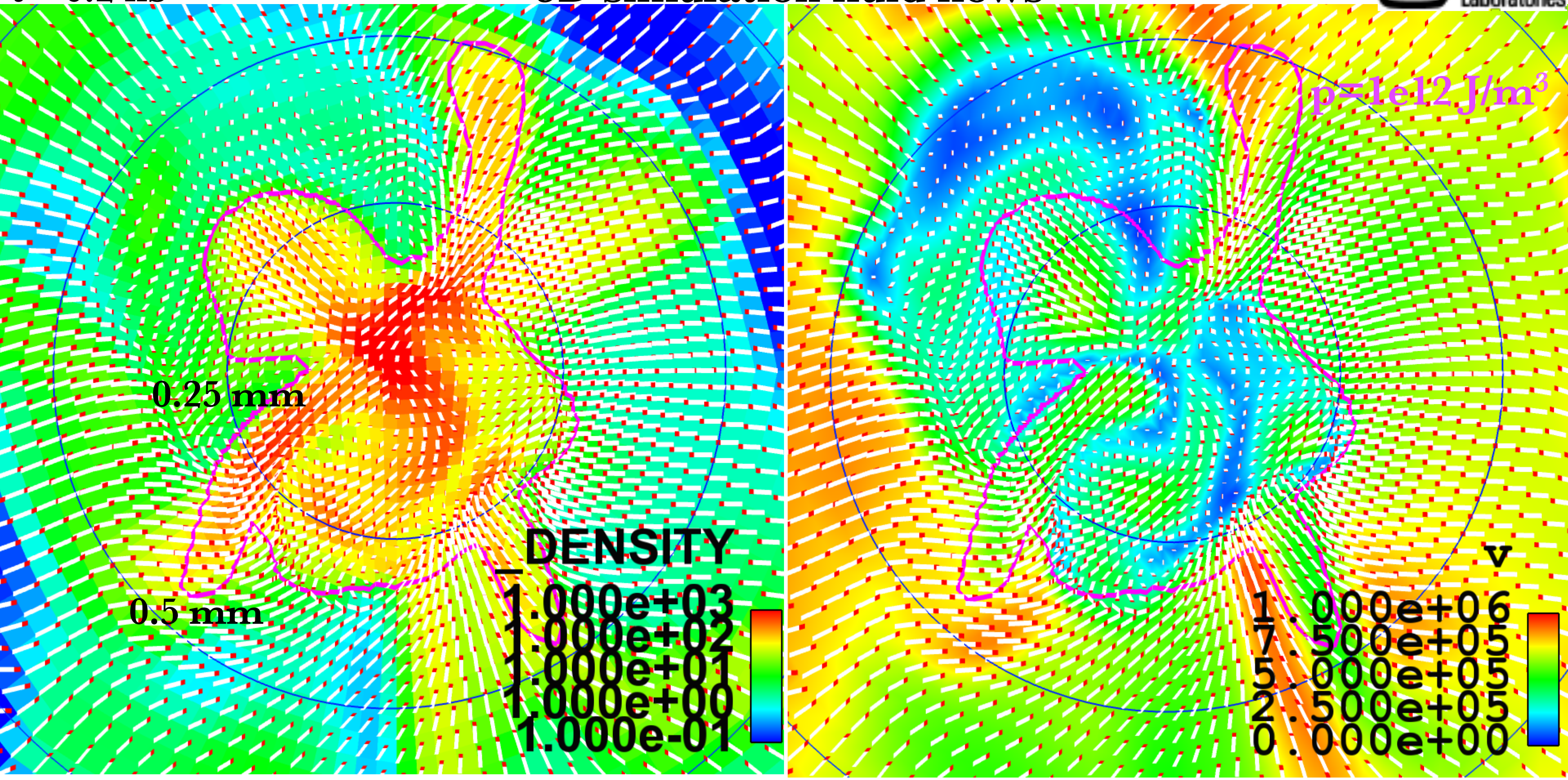


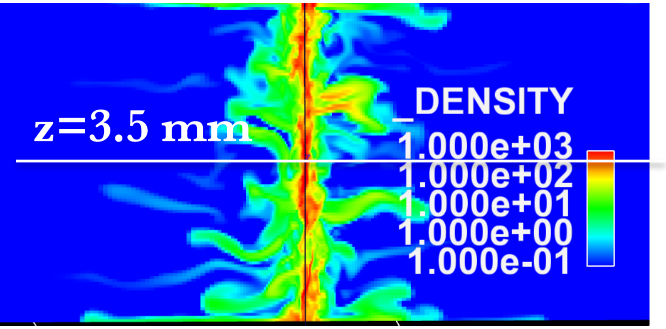


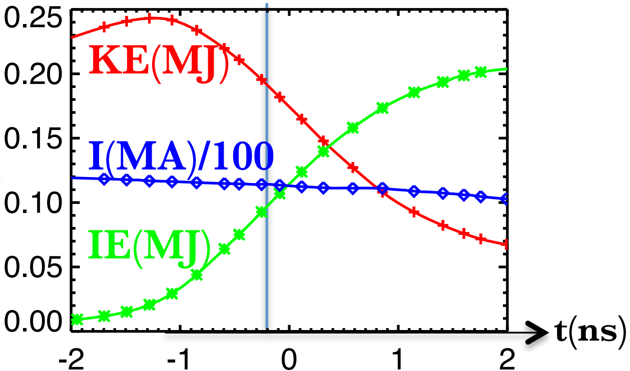


t=-0.2 ns

3D simulation fluid flows

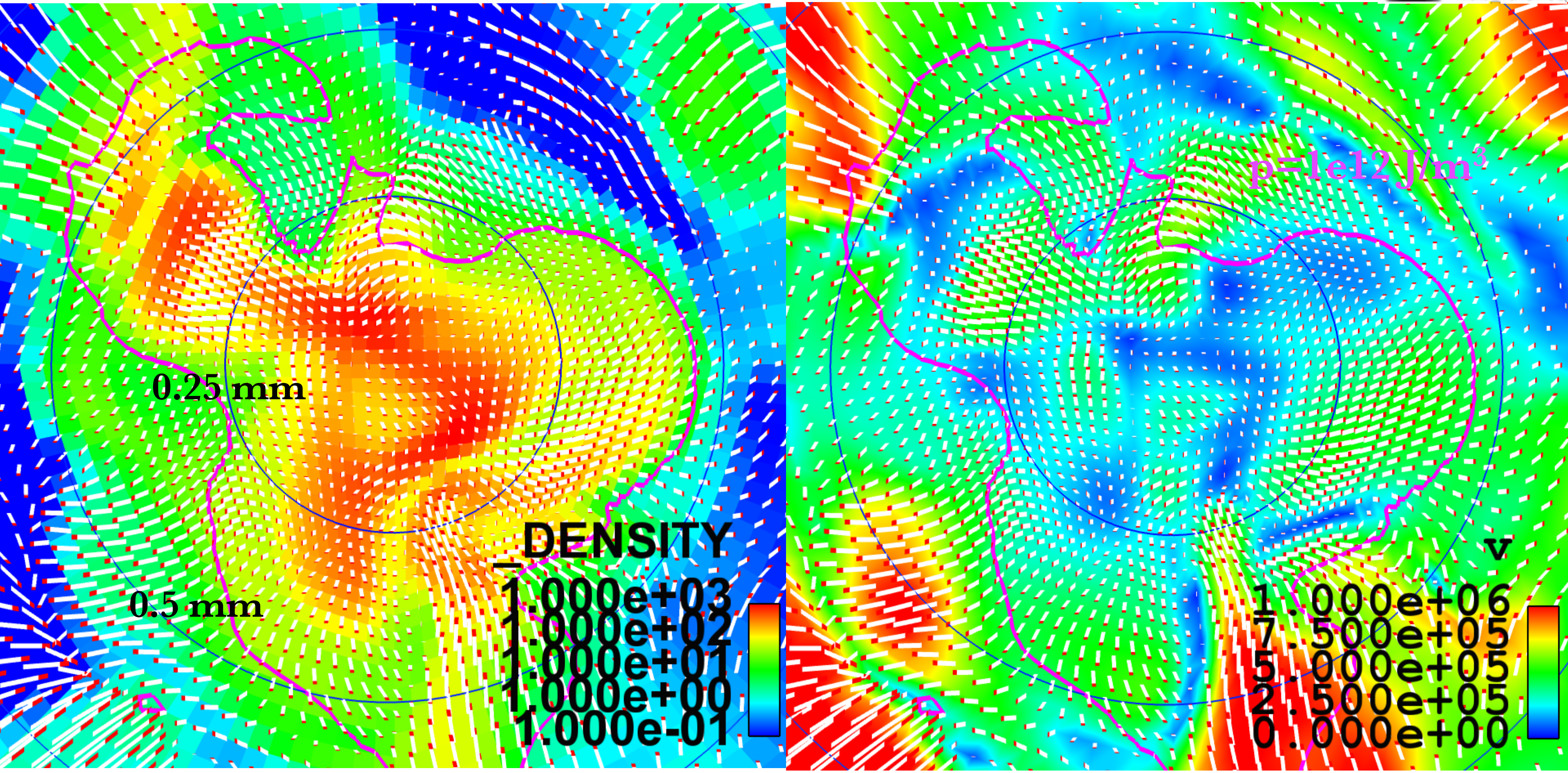


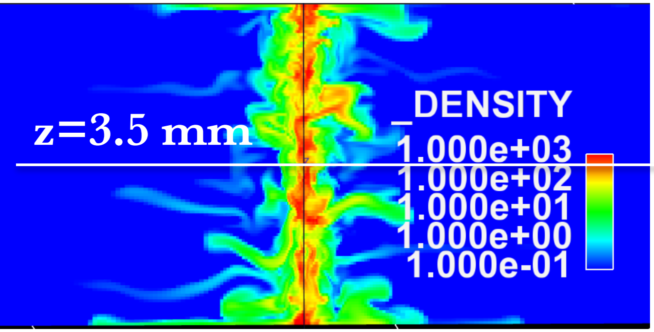


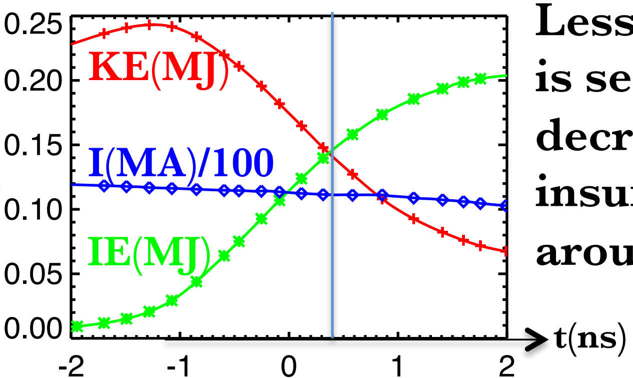


t=0.4 ns

3D simulation fluid flows



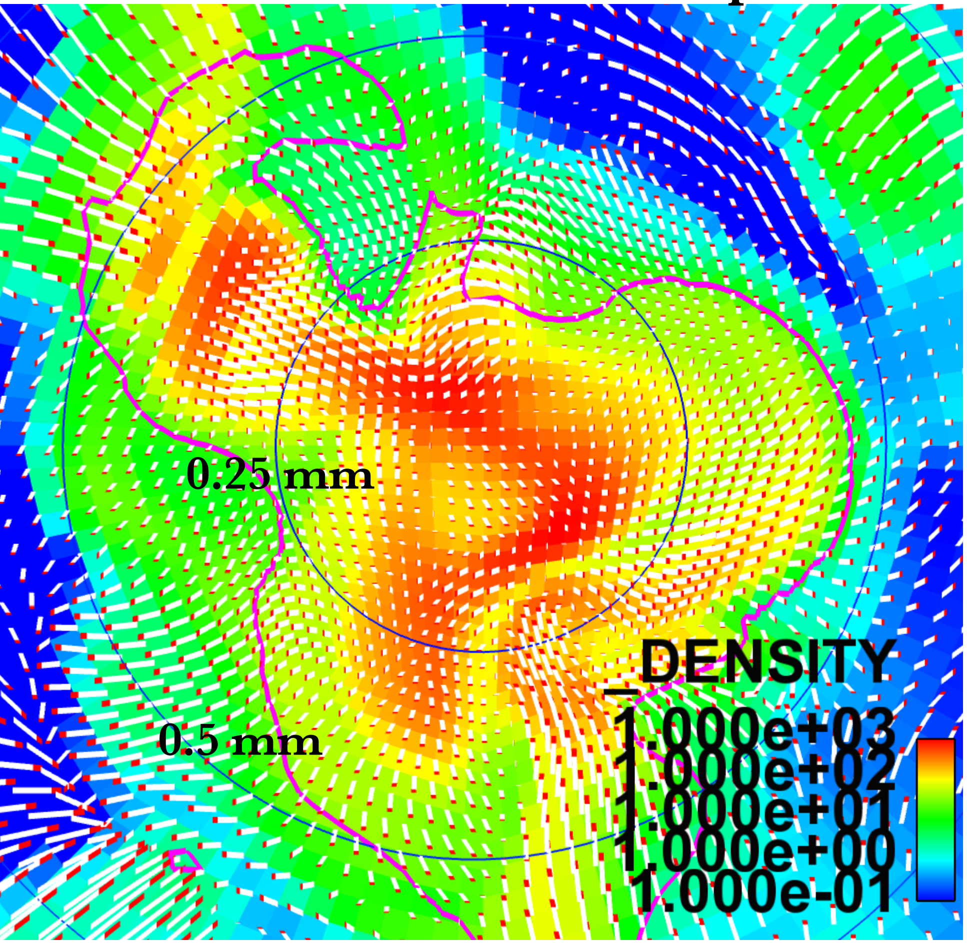


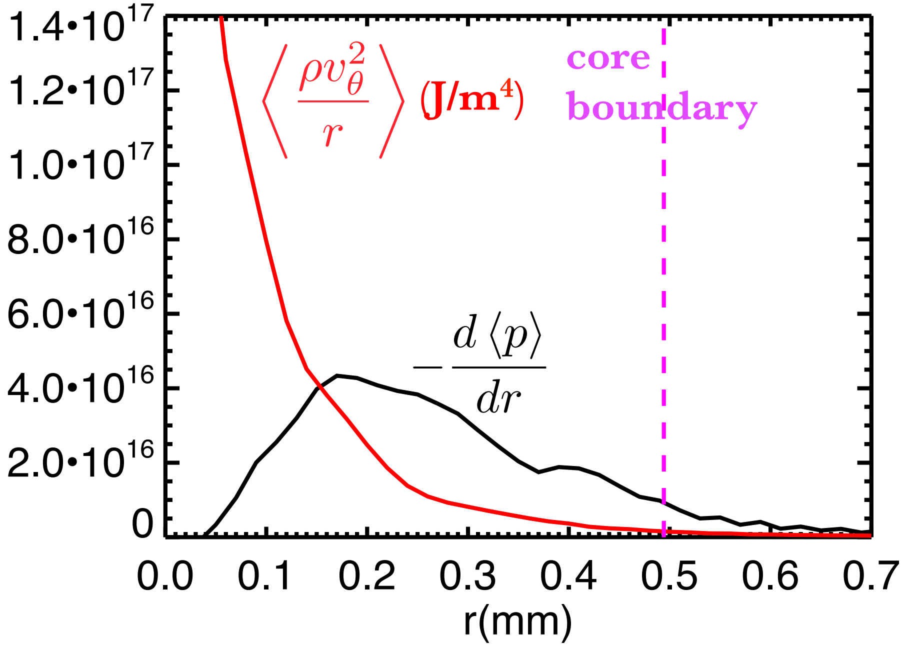


Less evidence for vortices. This is sensible: we are sampling decreasing p_{ram}, which is insufficient to “turn the flow around”.

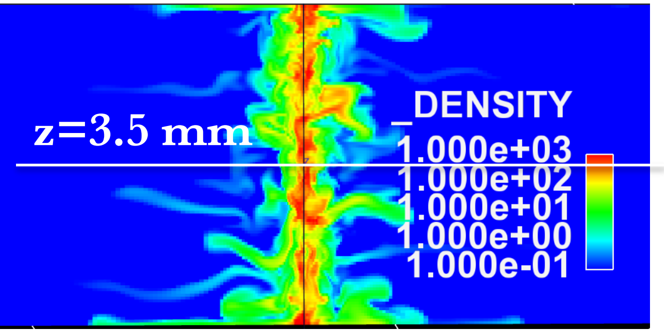
t=0.4 ns

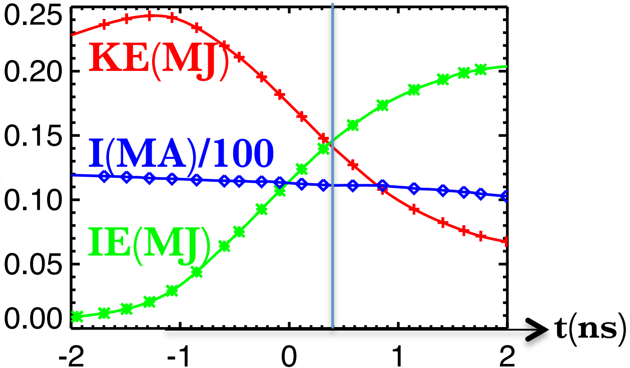
Back up: centrifugal force for t>0





Now centrifugal force << pressure gradient over much of the core.





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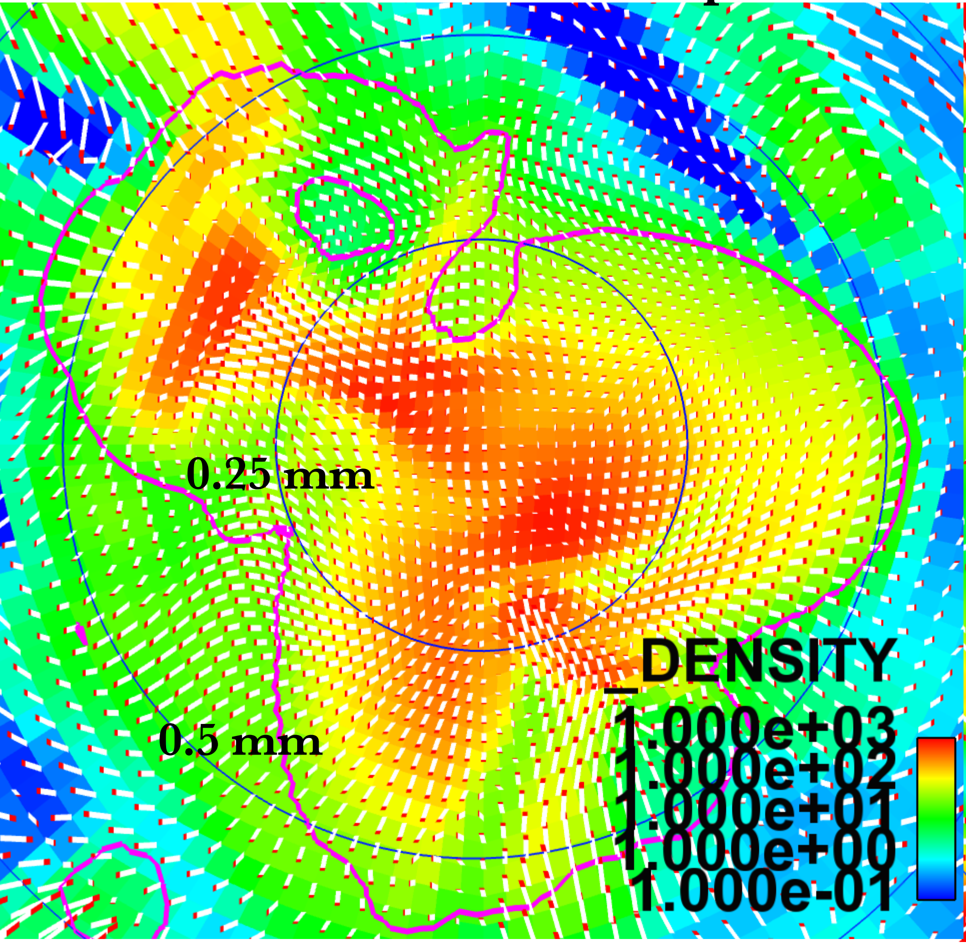
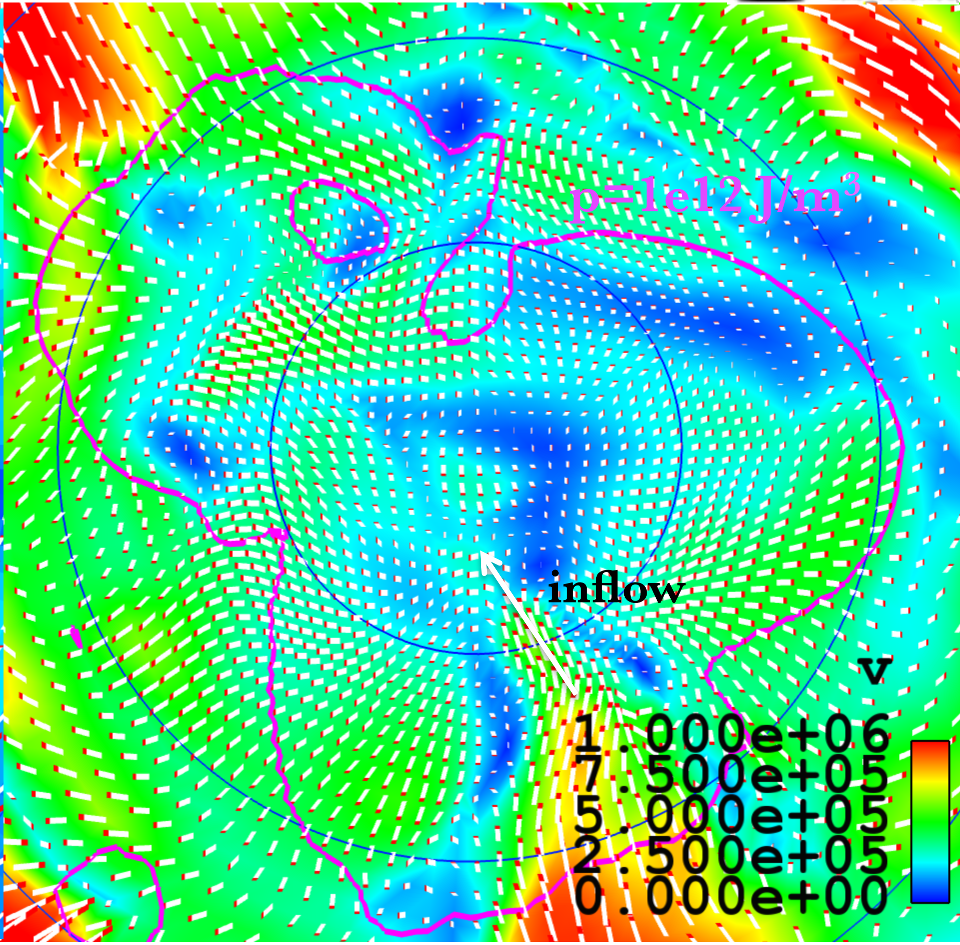
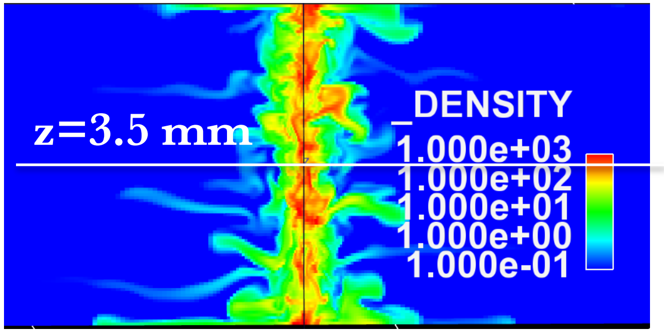
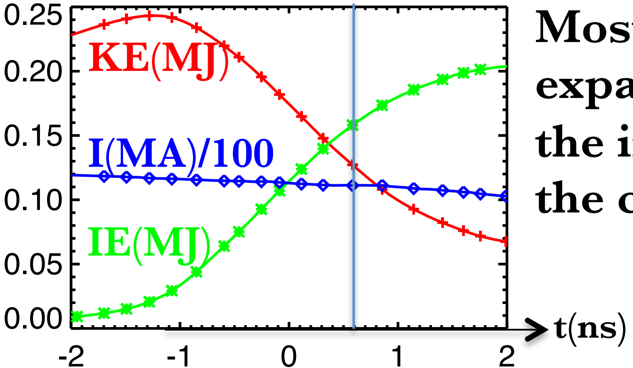
t=0.6 ns

Backup: 3D simulation fluid flows



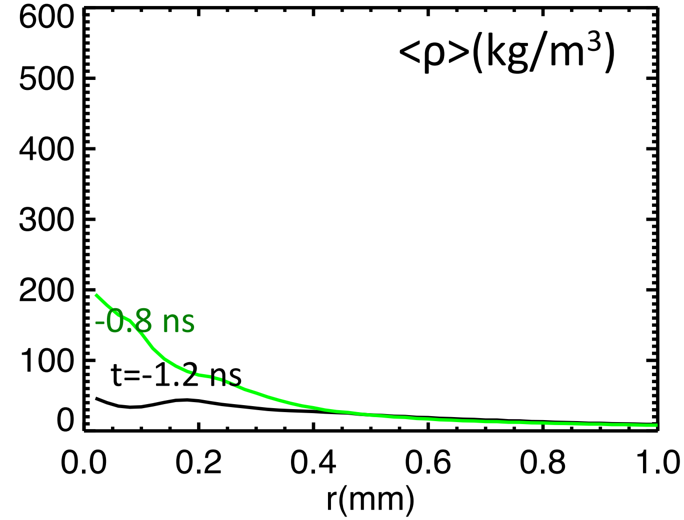


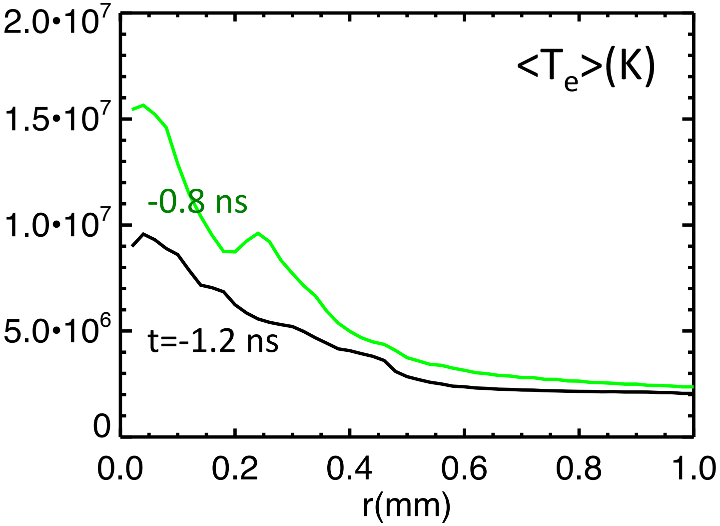




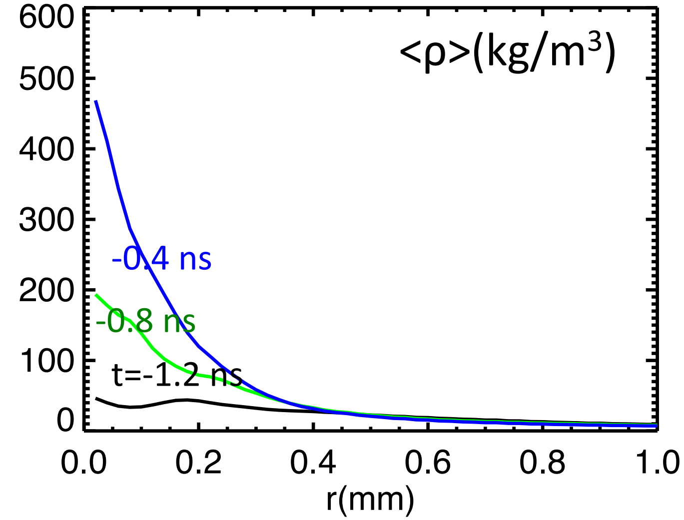
Most of the core plasma is expanding outward, but notice the inflow of hot plasma into the core.

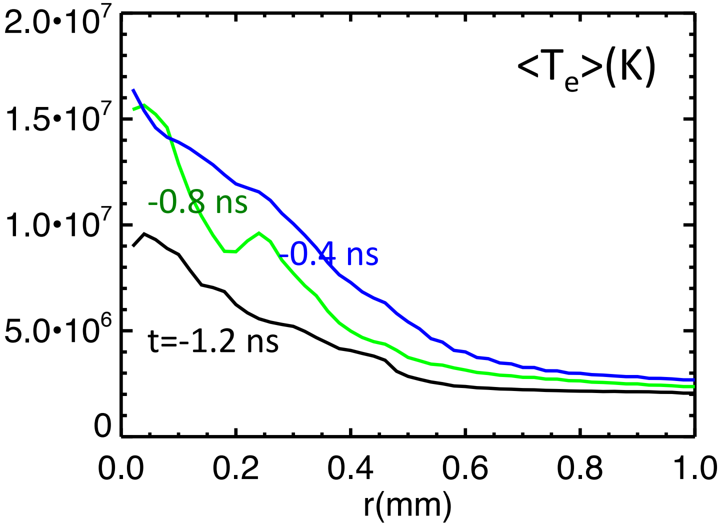
Effect of residual flows on energy transport



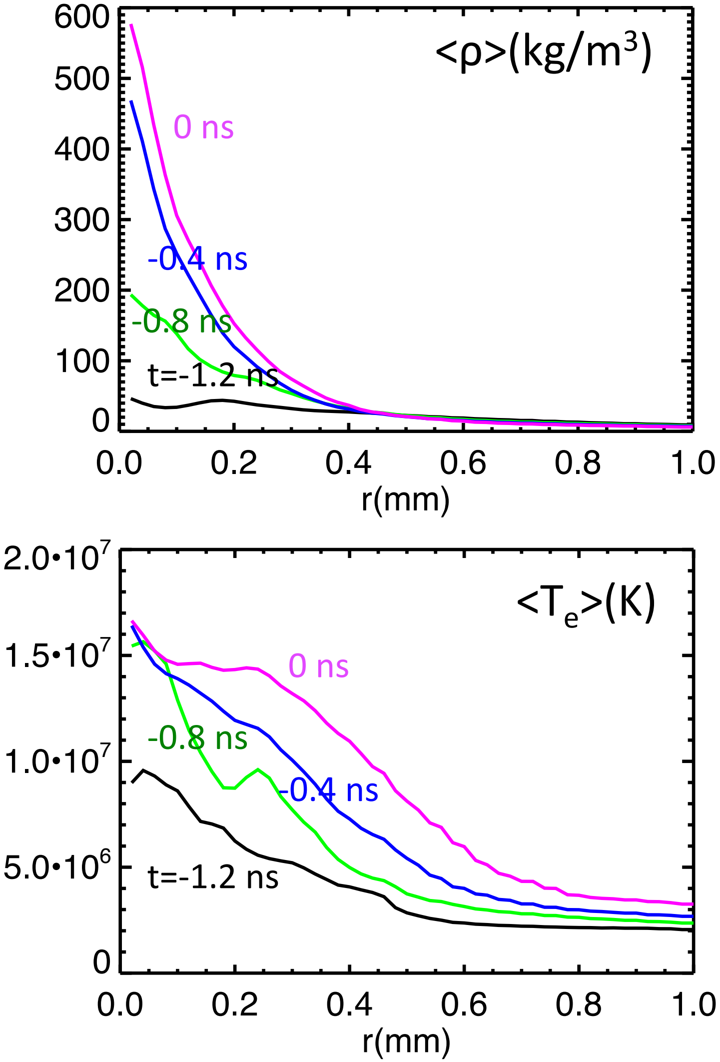


Effect of residual flows on energy transport





On-axis T_e is constant during compression



After t=-1 ns, T_e(r=0) doesn't increase--not good for HED/ICF.

Usually, plasma heats while compressing
i.e. for adiabatic plasma

$$p \sim \rho^{\gamma}$$

$$p = \frac{k_b}{m_i}\rho T \Rightarrow T \sim \rho^{\gamma-1}$$

γ~1.3 for W

T↑ as ρ↑

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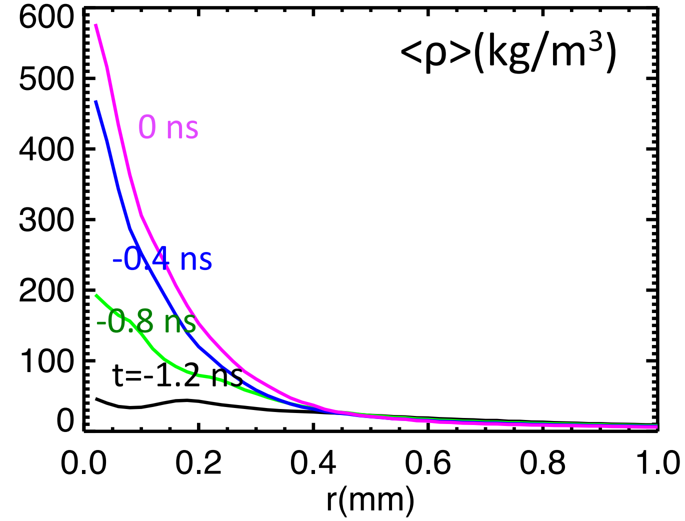

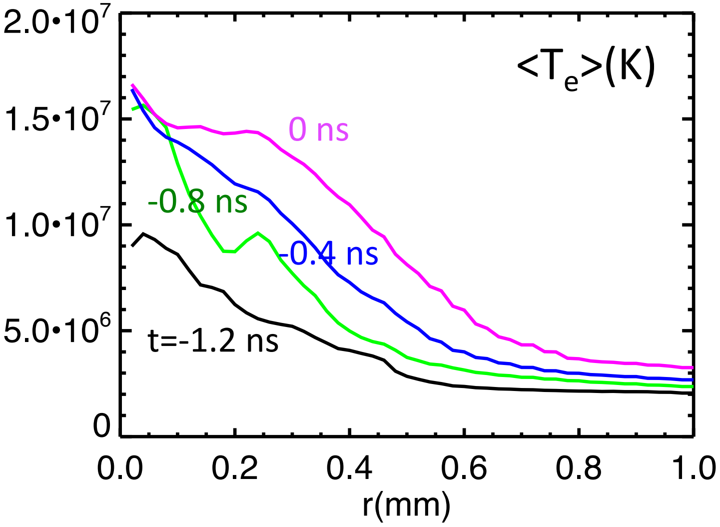

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$$p \sim \rho^{\gamma}$$

$$p = \frac{k_b}{m_i}\rho T \Rightarrow T \sim \rho^{\gamma - 1}$$

γ~1.3 for W

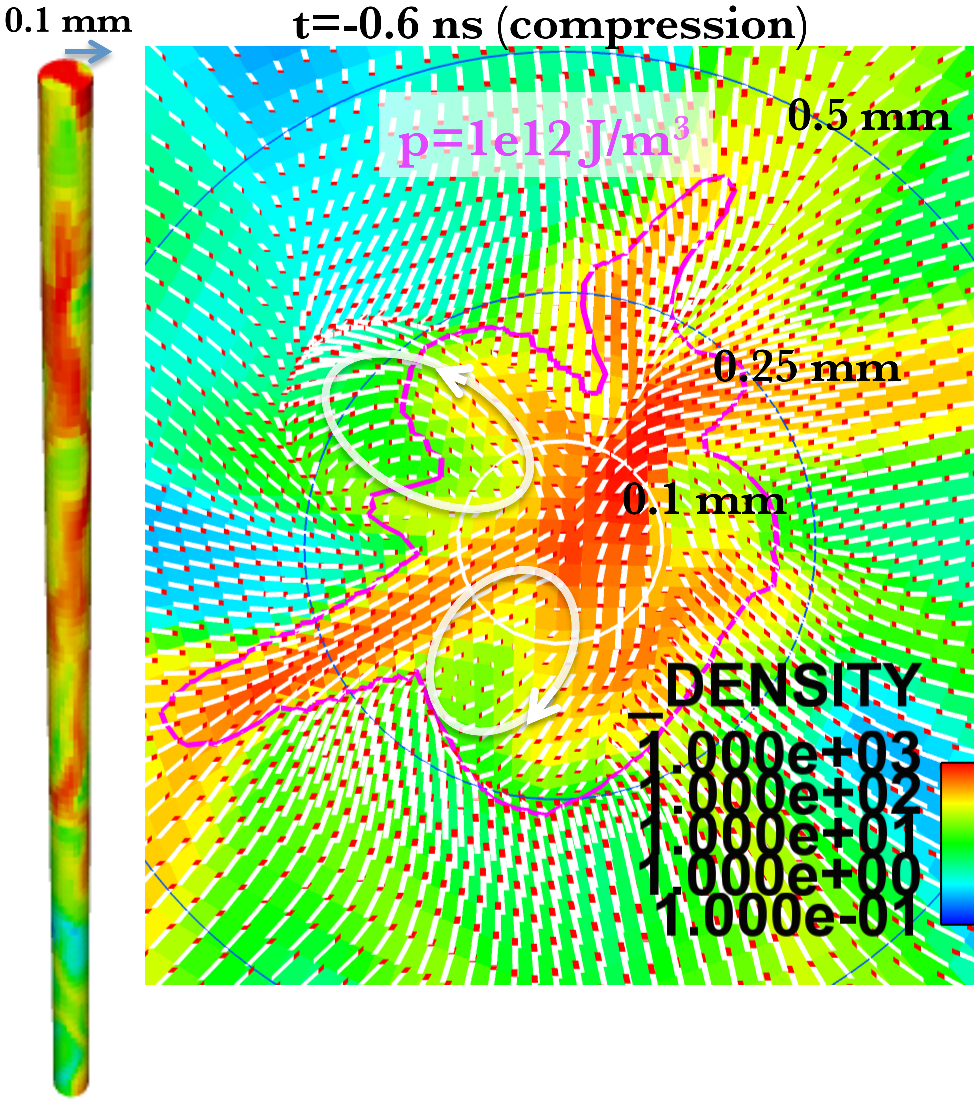
T↑ as ρ↑

Of course, plasma is not adiabatic; thermal conduction could prevent T from rising (recall radiation is off).

But a 1D estimate shows thermal conduction is *too weak* an effect.

Could convective flows explain this behavior?

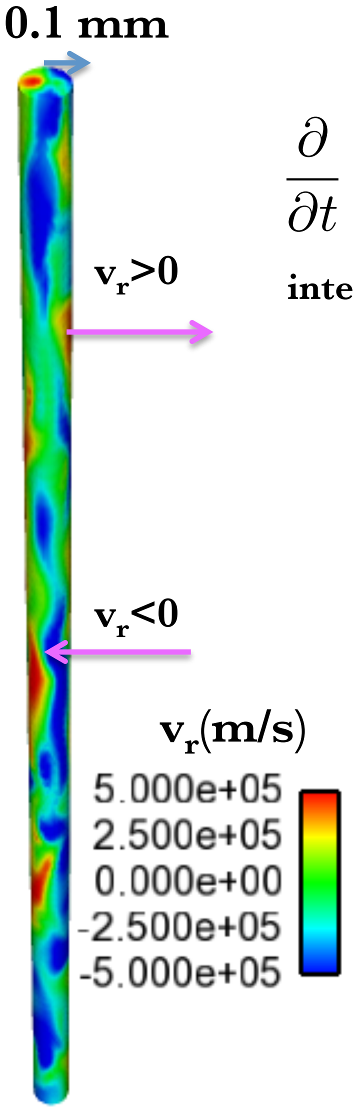
Effect of convection on energy transport



During compression, we saw outflow of hot core plasma – cooling mechanism.

We apply the energy equation to a R=0.1 mm cylinder to study these convective effects.

Convective energy flow dominates during compression



Energy convection in (source): -1.23e13 W

Energy convection out (sink): 9.96e12 W

$$\frac{\partial}{\partial t}\int_V \underbrace{\rho e dV}_{\text{internal energy}} = -\int_{v_r<0} \rho e \mathbf{v}\cdot d\mathbf{S} - \int_{v_r>0} \rho e \mathbf{v}\cdot d\mathbf{S}$$

pdV heating (source): -6.85e12 W

pdV cooling (sink): 2.52e12 W

$$-\int_{\nabla\cdot v<0} p\nabla\cdot\mathbf{v}dV - \int_{\nabla\cdot v>0} p\nabla\cdot\mathbf{v}dV$$

shock heating (source): 1.95e12 W

Joule heating (source): 1.7e11

thermal conduction (sink): 3.2e10 W

$$+\int_V \tau : \nabla\mathbf{v}dV + \int_V \frac{j^2}{\sigma}dV - \int_S \mathbf{q}\cdot d\mathbf{S}$$

- Even during compression, convection of energy OUT is large. Combined with pdV cooling, it is responsible for keeping T(r=0)~const.
- Convection leads to enhanced thermal transport over thermal conduction
- However κ in DT is larger: $\kappa \propto \dfrac{T^{5/2}}{Z\ln\Lambda}$

Volume "V"
Surface "S"
t=-0.6 ns

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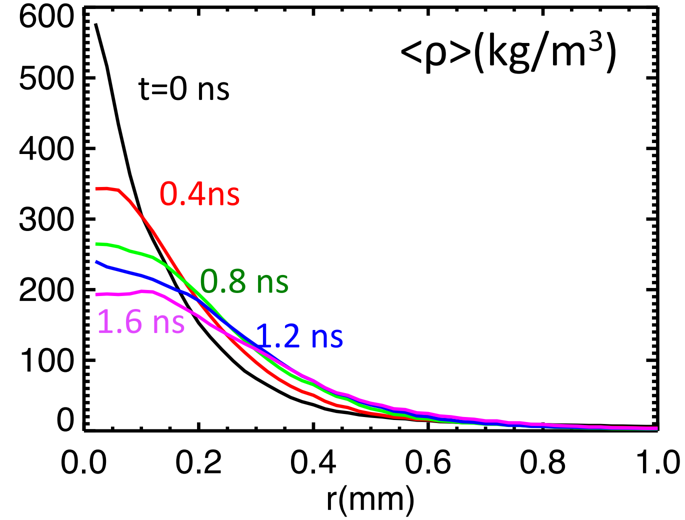
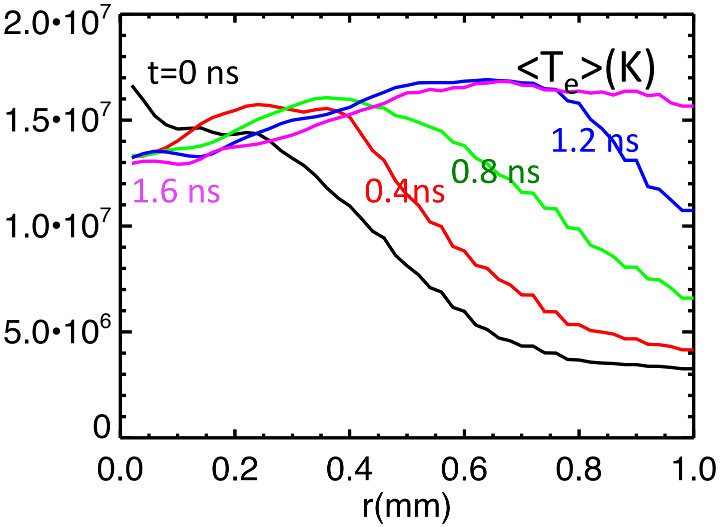
Backup: on-axis T_e is constant during expansion





After t=0.4 ns, $T_e(r=0)$ doesn't decrease.

Usually, plasma cools while expanding
i.e. for adiabatic plasma

$$T \sim \rho^{\gamma-1} \qquad \gamma\text{~}1.3 \text{ for W}$$

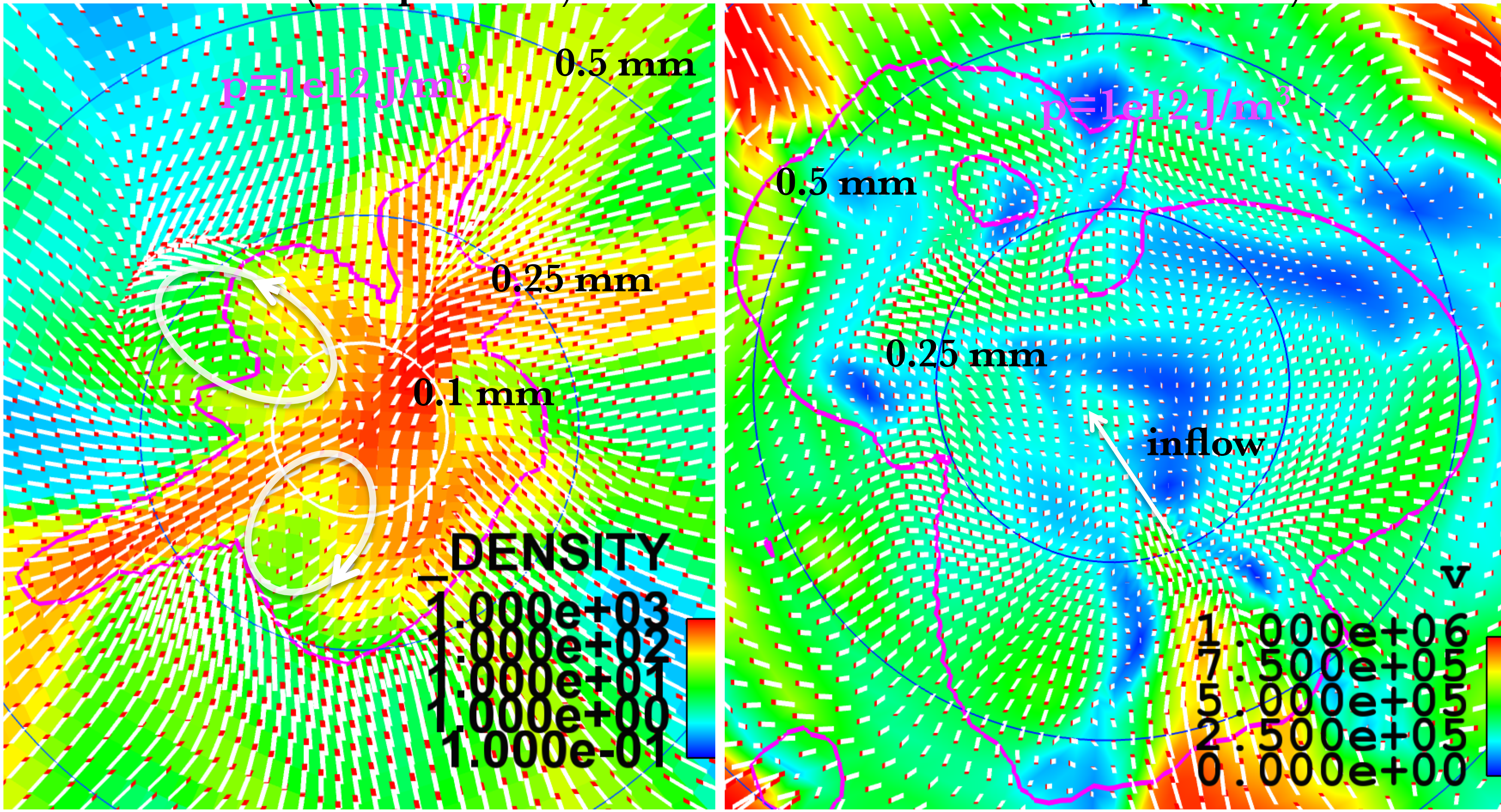
T↓ as ρ↓

Plasma is not adiabatic. Joule heating, thermal conduction, shock heating could all provide heating.

But again, estimates show these effects are *too weak* to counteract pdV cooling.

Could convective flows explain this behavior?

Backup: effect of convection on energy transport

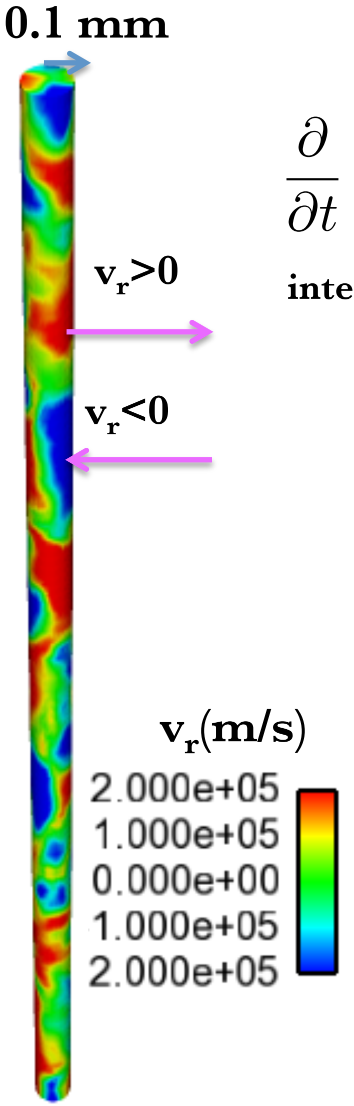

t=-0.6 ns (compression)

t=0.6 ns (expansion)

During compression, we saw outflow of hot core plasma – cooling mechanism. During expansion, we see inflow of hot material – heating mechanism

We apply the energy equation to a R=0.1 mm cylinder to study these convective effects.

Backup: Convective energy flux dominates during expansion



Energy convection out (sink): -1.17e13 W

Energy convection in (source): 7.7e12 W

$$\frac{\partial}{\partial t}\int_V \underbrace{\rho e dV}_{\text{internal energy}} = -\int_{v_r>0}\rho e\mathbf{v}\cdot d\mathbf{S} - \int_{v_r<0}\rho e\mathbf{v}\cdot d\mathbf{S}$$

pdV cooling (sink): -2.65e12 W

pdV heating (source): 1.5e12 W

$$-\int_{\nabla\cdot v>0} p\nabla\cdot\mathbf{v}dV - \int_{\nabla\cdot v<0} p\nabla\cdot\mathbf{v}dV$$

shock heating (source): 1.16e11 W

Joule heating (source): 6.9e10 W

thermal conduction (source): 3e10 W

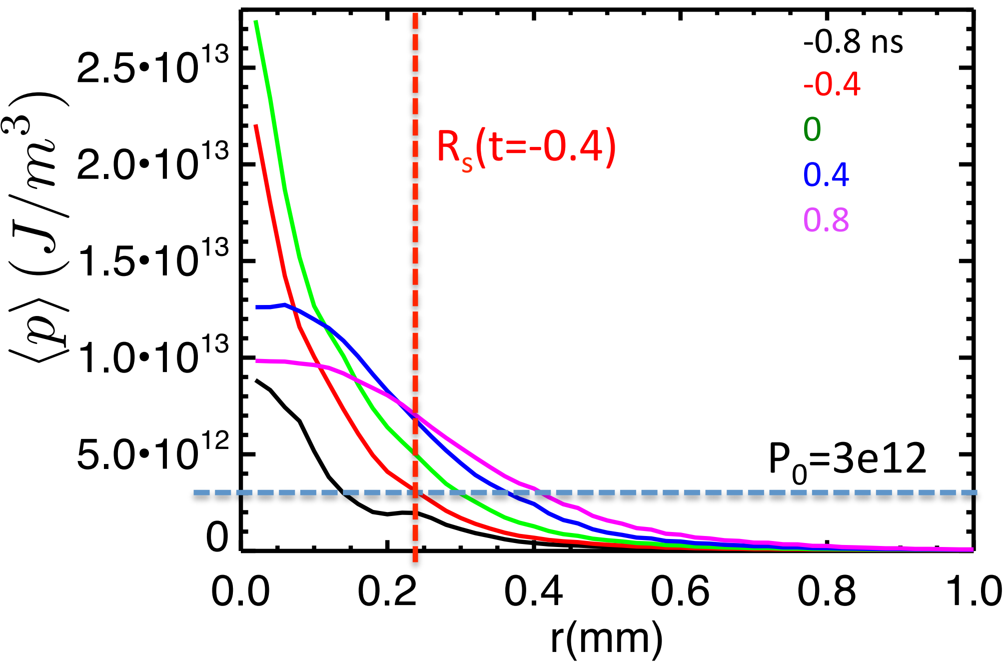
$$+\int_V \tau:\nabla\mathbf{v}dV + \int_V \frac{j^2}{\sigma}dV - \int_S \mathbf{q}\cdot d\mathbf{S}$$

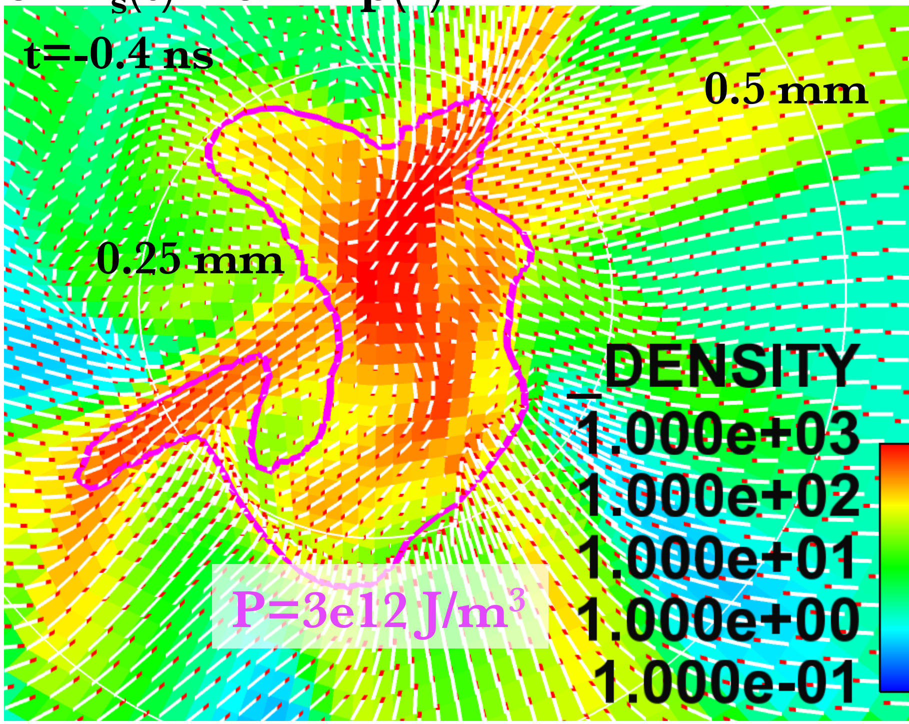
-Even though plasma is expanding, inflow transports significant energy into the core. This is the reason $T_e(r=0)$ can stay constant during expansion

-Convection leads to enhanced thermal transport over thermal conduction

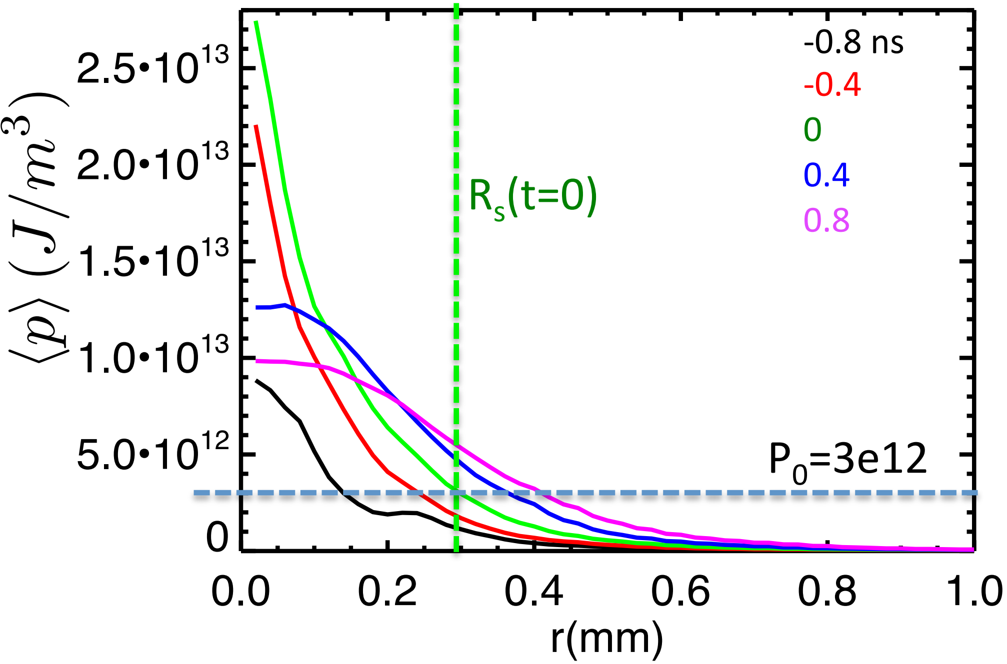
Volume "V"
Surface "S"
t=0.6 ns

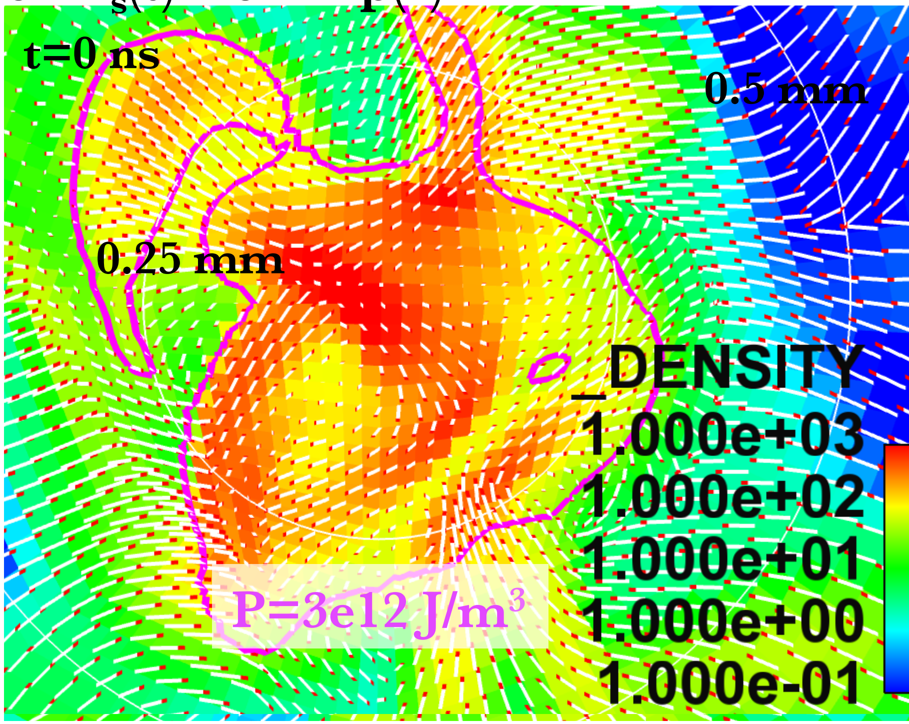
Backup: determination of $R_s(t)$ from <p(r)>

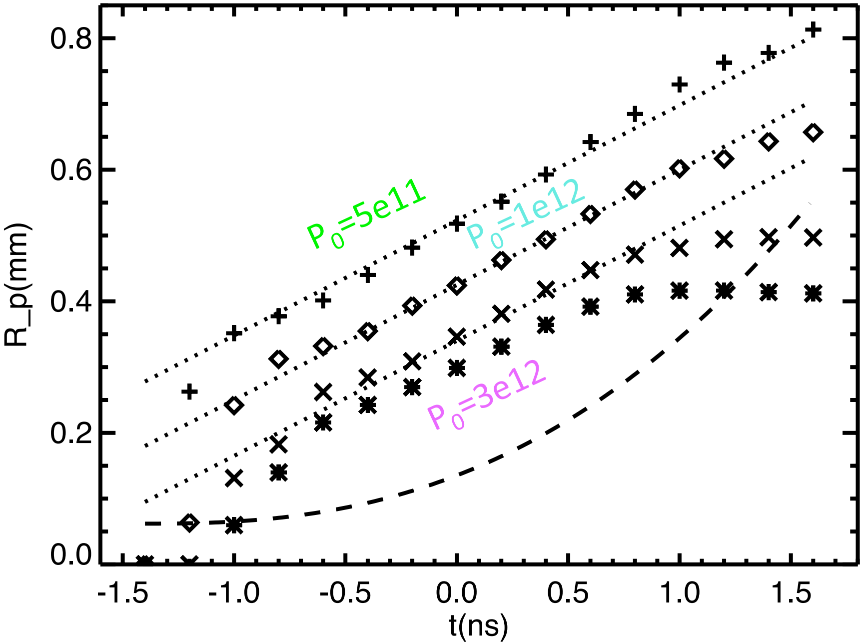




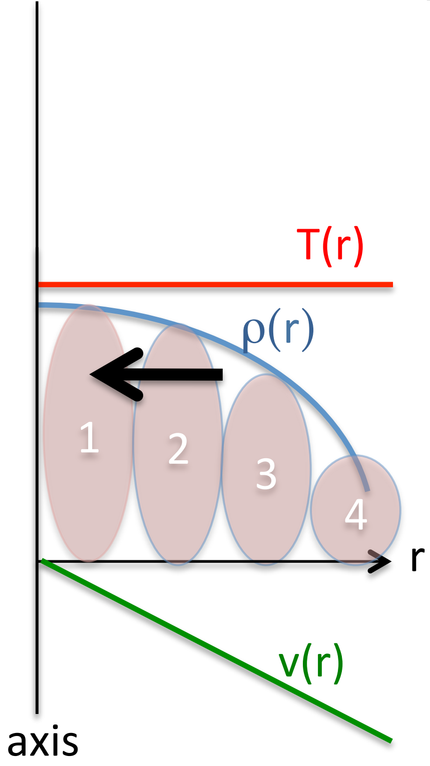
Backup: determination of $R_s(t)$ from <p(r)>

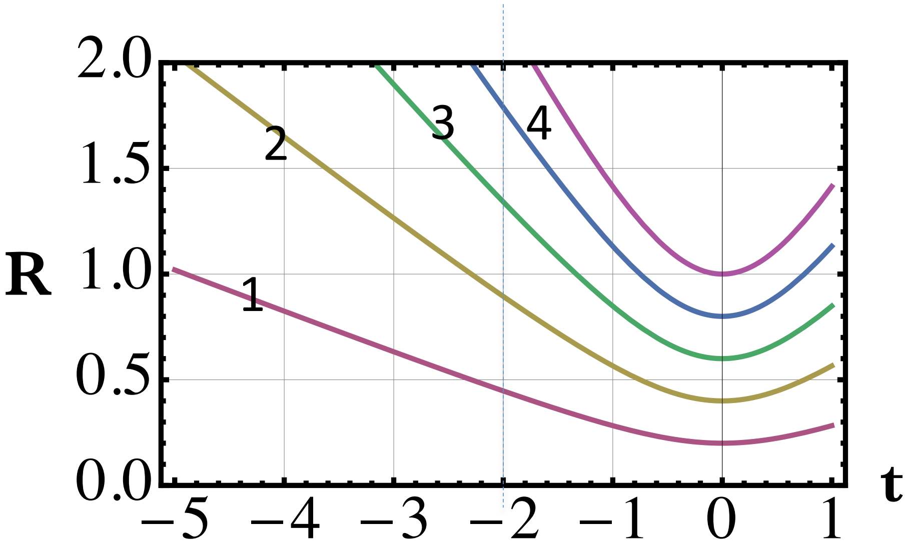






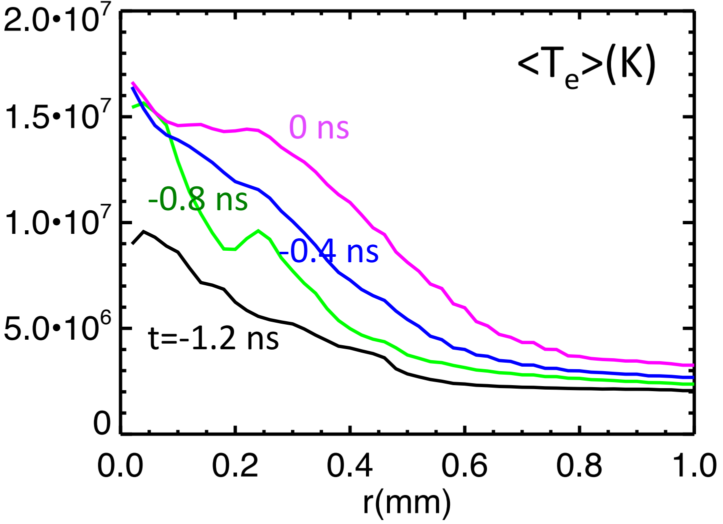
Backup: Justification for isothermal solution



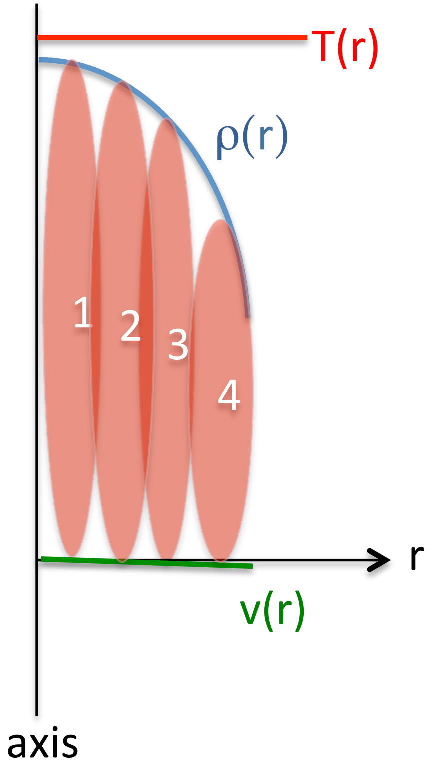


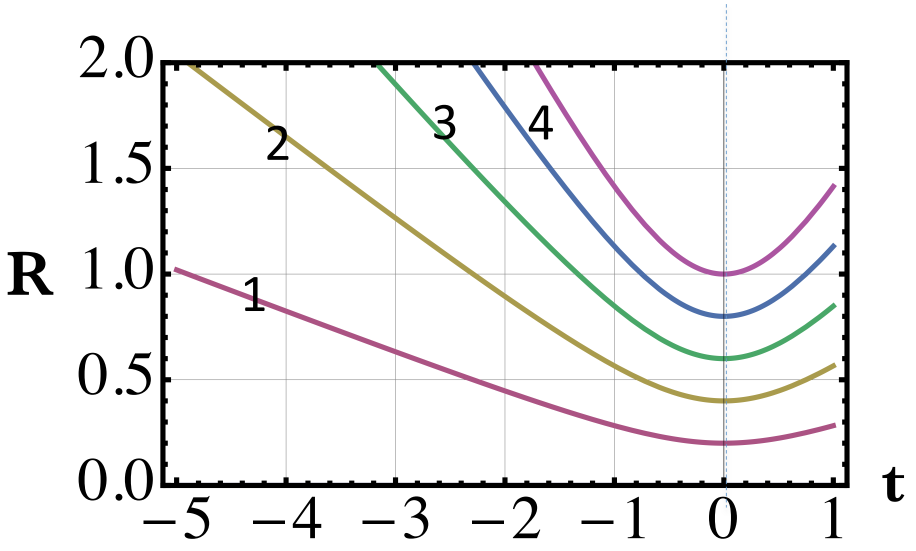
v(r) is linear. This allows particles to compress in unison, without shocks.

Also now finite T is allowed. We consider the isothermal solution, motivated by the enhanced thermal transport seen in 3D simulation.


3D Simulation

Backup: Effect of ε on compression





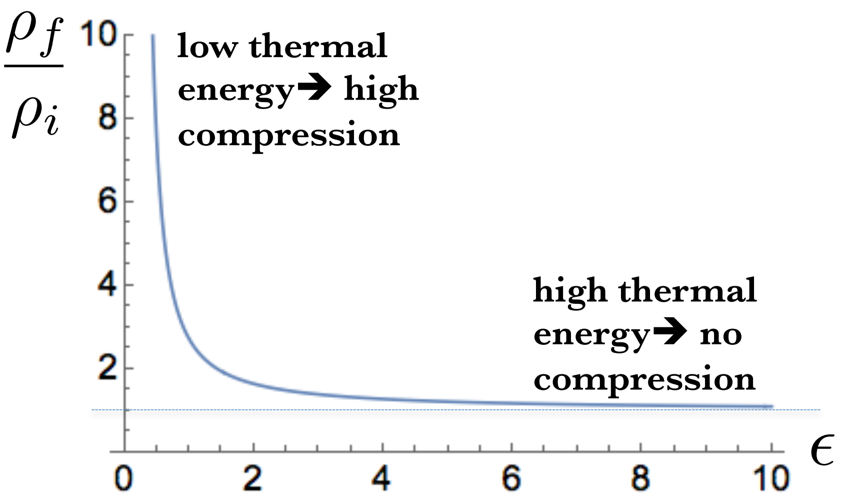
Stagnation is complete. All kinetic energy has converted to internal energy. The key parameter:

$$\epsilon = \frac{\int pdV}{\int \frac{1}{2}\rho v^2 dV}$$

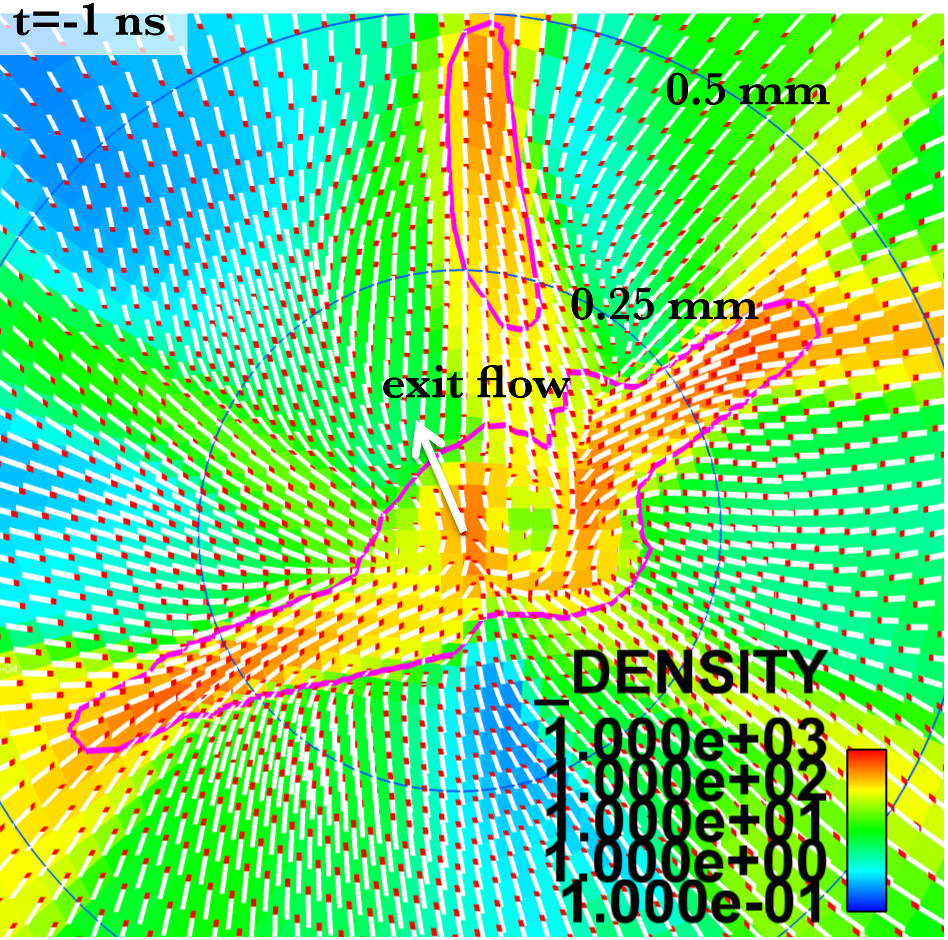
Initial thermal energy

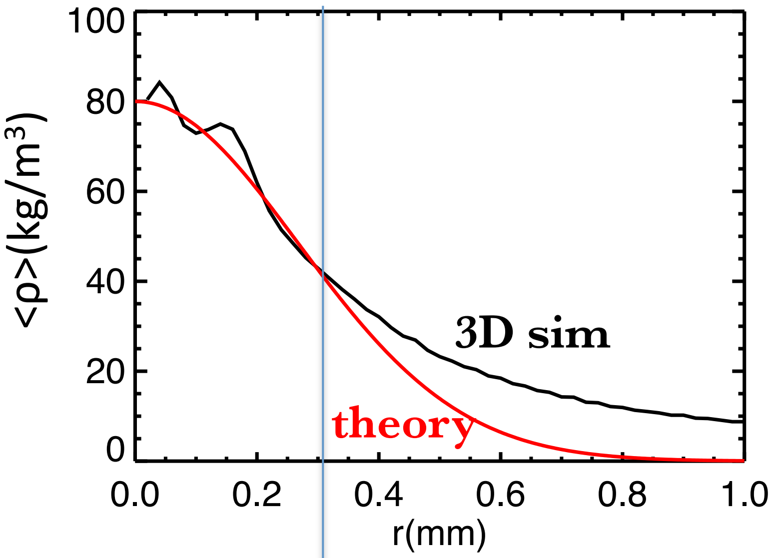
Initial kinetic energy

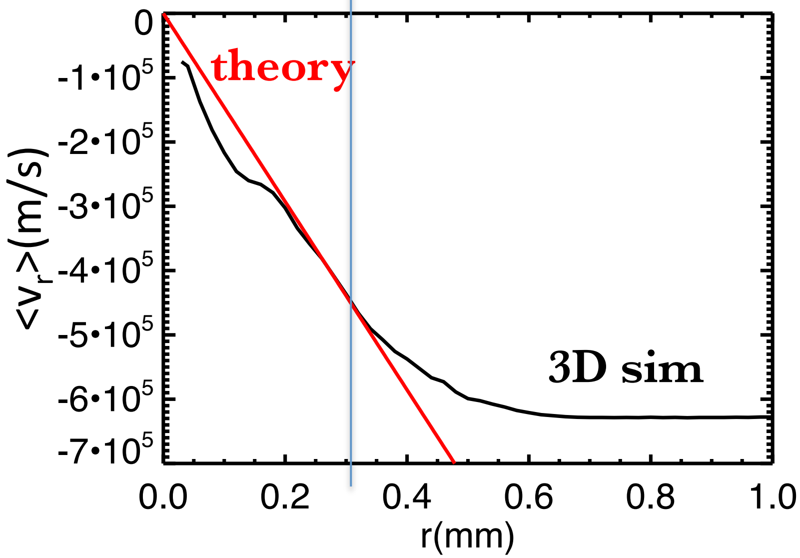
$$\frac{\rho_f}{\rho_i} = e^{1/\epsilon}$$

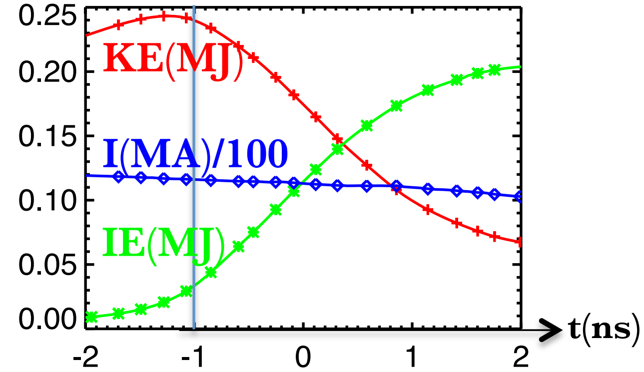


Backup: profile comparison between shockless solution and 3D simulation









3D profiles agree with theoretical fits for r≤0.3 mm

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Backup: energy flow in a Z pinch